Understanding the Economic Determinants of the Severity of Operational Losses: A regularized Generalized Pareto Regression Approach
-
Supplementary Material

1 Simulation study - additional simulations

In this section, we present the results of an additional simulation study, with the goal to emphasize the usefulness of the proposed regularization in a realistic framework. To do so, we combine the different realistic models used in the simulations of Subsection 4.2 into a single simulation study. We consider the case where $n = 4,000$ (as in Subsections 4.1 and 4.2) and where $n = 10,000$ (as in our empirical application). The data generating process is the following (using the notation developed in Section 3):

$$Y_{t,i} \sim GPD(Y_{t,i}; \gamma(x^{\gamma}_{t,i}), \sigma(x^{\sigma}_{t,i})), \tag{1}$$

$$\log(\gamma(\boldsymbol{x}^{\gamma}_{t,i})) = \alpha^{\gamma}_0 + \sum_{l=1}^{p_\gamma} \alpha^{\gamma}_l x^{\gamma}_{t,i}(l), \tag{2}$$

$$\log(\sigma(\boldsymbol{x}^{\sigma}_{t,i})) = \alpha^{\sigma}_0 + \sum_{l=1}^{p_\sigma} \alpha^{\sigma}_l x^{\sigma}_{t,i}(l), \tag{3}$$

with $p_\gamma = p_\sigma = p = 50$, $x^{\sigma}_{t,i}(l) = x^{\gamma}_{t,i}(l) = x_{t,i}(l)$, $\forall\, t, i, l$ (imposing collinearity of the covariates across distribution parameters) and $\alpha^{\gamma} = (-.9; -.3; .2; .2)$, $\alpha^{\sigma} = (4; .6; .4; -.3)$. For the different $x_{t,i}(l)$, we use two time series models:

$$x_{t,i}(l) = x_{t-1,i}(l) + \epsilon_{t,i}(l),\ l = 1, 11, \ldots, 20, \tag{4}$$

$$x_{t,i}(l) = 0.7x_{t-1,i}(l) + \varepsilon_{t,i}(l),\ l = 2, 3, 21, \ldots, 50, \tag{5}$$

where $\boldsymbol{\epsilon}$ and $\boldsymbol{\varepsilon}$ follow (distinct) multivariate standard t distributions with 6 degrees of freedom ($MVT(6, \Sigma)$). The covariance matrix Σ takes value 1 on the diagonal elements, 0.5 everywhere else. This set-up combines the features of all simulations done in Subsection 4.2., inducing time-series stationary, non-stationary, non-Gaussian and collinearity effects. As in Subsection 4.2, $n = \sum_{t=1}^{T} n_t$. Firstly, we assume $n_t = 1$, $\forall t$, and $T = n \in \{4,000; 10,000\}$. Detailed results are presented in Table 1, upper panel (Alternative Model Mixed I). As in 4.2, we observe bad results for the selection strategy based on p-values, whereas the BIC_2 for smoothing parameter selection seems to provide the best results, with the adLASSO penalty exhibiting slightly better RMSE and CCR. We do not observe much difference between $n = 4,000$ and $n = 10,000$ in term of RMSE. This might be due to the fact that with non-stationary time series, the variance of some covariates growth to infinity with time, impacting the estimation process (thus, longer time series provide more variables estimates). However, in term of t.p. rate for γ, we see a strong improvement: from 56.7% (resp. 77.2%) to 94.7% (resp. 97%) for LASSO (resp. adLASSO).

Secondly, we also consider the case where $n_t \neq 1$ and T small. Indeed, in practice we observe several losses during a given time period, and only short time series. Therefore we assume $n_t \in \{20; 80; 200\}$, $\forall t$, and $T = 50$, such that we obtain final sample sizes $n \in \{1,000; 4,000; 10,000\}$. Results are presented in Table 1, lower panel (Alternative model mixed II). Conclusions regarding p-values-based technique are similar. However, in this configuration, LASSO provides equivalent or better results than adLASSO in term of RMSE and CCR. For $n = 1000$, it comes at the price of a low t.p. rate for γ, but when the sample size increases, t.p. becomes also better. In general, if the sizes of the effects of the informative covariates are small, we may identify them less frequently as such, but at least we prevent ourselves very well against false detections.

To conclude this section, it appears that, in the most realistic scenario Mixed II with 10,000 observations (i.e. the scenario that is the closest to the features of our dataset of operational losses), LASSO works best as well.

Alternative model, $p = 50$. $\alpha^{\gamma} = \{-.9; -.3; .2; .2\}$, $\alpha^{\sigma} = \{4; .6; .4; -.3\}$										
Model	Sample size	Penalty	Selection method	RMSE	RMSE(+)	$f.p.(\sigma)$	$t.p(\sigma)$	$f.p.(\gamma)$	$t.p.(\gamma)$	CCR
Mixed I	$n = 4000$	none	pval	1	0.781	0.336	1	0.342	0.938	0.686
		LASSO	AIC_1	0.135	0.299	0.416	1	0.43	0.953	0.609
			AIC_2	0.12	0.193	0.223	1	0.232	0.91	0.788
			BIC_1	0.123	0.137	0.142	1	0.154	0.892	0.861
			BIC_2	0.189	0.15	0.034	1	0.044	0.567	0.951
		adLASSO	AIC_1	0.148	0.337	0.21	1	0.2	0.907	0.808
			AIC_2	0.117	0.23	0.105	1	0.133	0.883	0.887
			BIC_1	0.085	0.106	0.033	1	0.059	0.867	0.954
			BIC_2	0.096	0.092	0.004	1	0.027	0.772	0.979
	$n = 10000$	none	p-val.	1	0.817	0.326	1	0.327	0.993	0.7
		LASSO	AIC_1	0.149	0.302	0.358	1	0.337	0.997	0.68
			AIC_2	0.153	0.224	0.258	1	0.226	0.995	0.777
			BIC_1	0.171	0.159	0.123	1	0.15	0.993	0.874
			BIC_2	0.238	0.131	0.045	1	0.091	0.947	0.936
		adLASSO	AIC_1	0.26	0.583	0.347	1	0.357	0.988	0.676
			AIC_2	0.201	0.35	0.158	1	0.185	0.983	0.842
			BIC_1	0.086	0.094	0.014	1	0.034	0.98	0.977
			BIC_2	0.098	0.079	0.001	1	0.018	0.97	0.99
Mixed II	$n = 1000$	none	pval	1	1.046	0.893	0.915	0.906	0.943	0.168
		LASSO	AIC_1	0.004	0.026	0.366	0.992	0.299	0.578	0.681
			AIC_2	0.003	0.006	0.208	0.983	0.115	0.387	0.833
			BIC_1	0.002	0.003	0.143	0.983	0.095	0.395	0.872
			BIC_2	0.002	0.002	0.066	0.922	0.018	0.16	0.934
		adLASSO	AIC_1	0.01	0.044	0.388	0.872	0.299	0.492	0.665
			AIC_2	0.012	0.036	0.339	0.86	0.271	0.465	0.699
			BIC_1	0.005	0.016	0.244	0.823	0.248	0.457	0.752
			BIC_2	0.006	0.016	0.244	0.827	0.24	0.44	0.756
	$n = 4000$	none	p-val.	1	1.058	0.873	0.97	0.91	0.925	0.177
		LASSO	AIC_1	0.004	0.009	0.349	1	0.312	0.875	0.693
			AIC_2	0.003	0.006	0.216	1	0.209	0.832	0.8
			BIC_1	0.003	0.004	0.142	1	0.145	0.783	0.862
			BIC_2	0.005	0.004	0.059	0.998	0.05	0.465	0.934
		adLASSO	AIC_1	0.009	0.019	0.321	0.975	0.319	0.695	0.695
			AIC_2	0.009	0.015	0.23	0.968	0.223	0.618	0.779
			BIC_1	0.006	0.007	0.131	0.942	0.13	0.508	0.864
			BIC_2	0.007	0.007	0.086	0.925	0.096	0.412	0.897
	$n = 10000$	none	p-val.	1	1.051	0.868	0.99	0.881	0.918	0.193
		LASSO	AIC_1	0.006	0.011	0.357	1	0.238	0.988	0.726
			AIC_2	0.006	0.009	0.231	1	0.205	0.985	0.799
			BIC_1	0.007	0.006	0.139	1	0.157	0.962	0.863
			BIC_2	0.009	0.006	0.072	1	0.099	0.848	0.917
		adLASSO	AIC_1	0.023	0.047	0.266	0.988	0.355	0.848	0.71
			AIC_2	0.028	0.048	0.173	0.988	0.304	0.837	0.775
			BIC_1	0.014	0.015	0.084	0.982	0.139	0.663	0.887
			BIC_2	0.015	0.016	0.053	0.977	0.096	0.573	0.918

Table 1: Ratio of Mean Squared error (RMSE) between penalized and unpenalized estimates, false positive (f.p.) rate, true positive (t.p.) rate and correct classification rate (CCR) when the penalty is either LASSO or adLASSO. The lines *p-val.* relate to f.p. and t.p. rates when the selection is performed with Wald tests at the 5% test level. CCR is defined as the number of active covariates selected plus the number of uninformative covariates not selected, divided by the total number of covariates.

2 Additional results: adLASSO penalty

Using $\tau = Q_{0.75}$, the results obtained with the adLASSO penalty are given in Tables 2 and 3.

For Model 2, we select only the VIX for γ. Similarly to the LASSO results, we also select the Italian unemployment rate and the leverage ratio, but for the equation of σ. In addition, we select the short term (interbank) interest rate and the unemployment rate at the EU level. Using BIC_1 instead of BIC_2, we select in addition the consumer loan rate in the EU and the PRF.

For Model 3, we found a main effect of the Italian unemployment rate, and the VFTSE. Hence, Italian unemployment rate is again selected, whereas the VFTSE replaced the VIX. This is not surprising since VIX and VFTSE exhibit a very high correlation (around 0.96). Regarding the interactions, we select again EU GDP x CPBP. For these three variables, the signs of the regression coefficients are also the same. This time, however, we also select interactions between VFTSE and CPBP, as well as between EU unemployment rate and BDSF. The main effect of the GDP, and other interaction variables (especially related to HPI and DGR) are not selected. Using BIC_1 instead of BIC_2, we select numerous interaction variables, hardly interpretable. We see nevertheless that LR is selected, and enters the equation of γ with a negative regression coefficient, as for the LASSO solution.

In term of goodness-of-fit, Figures 1 and 2 show CV(CLS). For a censoring level beyond 0.8, the performance of all adLASSO models are worst than Model 3 obtained with LASSO and BIC_2. In particular, models with interactions are less good for all censoring levels.

Model 2, adLASSO						
BIC_2	$\hat{\alpha}^{\sigma}$	$\hat{\alpha}^{\sigma}_{+}$	(p-value)	$\hat{\alpha}^{\gamma}$	$\hat{\alpha}^{\gamma}_{+}$	(p-value)
(Intercept)	10.004	10.007	(0.00)	-0.2	-0.206	(0.00)
IFRAUD	0.572	0.571	(0.00)	-0.018	-0.017	(0.33)
EFRAUD	0.588	0.588	(0.00)	-0.276	-0.274	(0.00)
EPWS	0.527	0.526	(0.00)	-0.108	-0.106	(0.00)
CPBP	1.104	1.104	(0.00)	-0.194	-0.193	(0.00)
BDSF	0.166	0.166	(0.00)	-0.043	-0.043	(0.02)
EDPM	0.723	0.725	(0.00)	-0.037	-0.038	(0.03)
Unemp. EU	0.278	0.463	(0.00)	-	-	-
Unemp. IT	-0.166	-0.243	(0.00)	-	-	-
LR	-0.021	-0.07	(0.00)	-	-	-
ST rate	0.162	0.317	(0.00)	-	-	-
VIX	-	-	-	0.027	0.065	(0.00)
BIC_1	$\hat{\alpha}^{\sigma}$	$\hat{\alpha}^{\sigma}_{+}$	(p-value)	$\hat{\alpha}^{\gamma}$	$\hat{\alpha}^{\gamma}_{+}$	(p-value)
(Intercept)	10.004	10.008	(0.00)	-0.201	-0.208	(0.00)
IFRAUD	0.572	0.569	(0.00)	-0.018	-0.017	(0.32)
EFRAUD	0.588	0.586	(0.00)	-0.276	-0.277	(0.00)
EPWS	0.527	0.525	(0.00)	-0.108	-0.107	(0.00)
CPBP	1.104	1.102	(0.00)	-0.193	-0.195	(0.00)
BDSF	0.166	0.164	(0.00)	-0.042	-0.043	(0.02)
EDPM	0.723	0.724	(0.00)	-0.037	-0.040	(0.02)
Unemp. EU	0.295	0.600	(0.00)	-	-	-
Unemp. IT	-0.171	-0.282	(0.00)	-	-	-
LOR EU	0.001	0.097	(0.00)	-	-	-
ST rates	0.175	0.318	(0.00)	-	-	-
VIX	-	-	-	0.037	0.068	(0.00)
PRF	0.002	0.054	(0.00)	-	-	-
LR	-0.024	-0.080	(0.00)	-	-	-

Table 2: Results of the regularized regressions for Model 2, using adLASSO penalties. Selection of the penalty parameters is performed over a grid ($[0.005; 0.01]$ for σ and $[0.001; 0.015]$ for γ) using either BIC_2 as a criterion (top, $\boldsymbol{\nu}_2 = (0.0054; 0.0055)$) or BIC_1 (bottom, $\boldsymbol{\nu}_1 = (0.0053; 0.0043)$.

Model 3, adLASSO						
BIC_2	$\hat{\alpha}^{\sigma}$	$\hat{\alpha}^{\sigma}_{+}$	(p-value)	$\hat{\alpha}^{\gamma}$	$\hat{\alpha}^{\gamma}_{+}$	(p-value)
(Intercept)	10.000	10.002	(0.00)	-0.194	-0.201	(0.00)
IFRAUD	0.573	0.573	(0.00)	-0.0199	-0.02	(0.25)
EFRAUD	0.59	0.589	(0.00)	-0.278	-0.278	(0.00)
EPWS	0.529	0.528	(0.00)	-0.109	-0.108	(0.00)
CPBP	1.101	1.091	(0.00)	-0.218	-0.293	(0.00)
BDSF	0.165	0.166	(0.00)	-0.044	-0.233	(0.00)
EDPM	0.722	0.721	(0.00)	-0.036	-0.035	(0.04)
Unemp. IT	-	-	-	-0.006	-0.046	(0.02)
VFTSE	-	-	-	0.007	0.024	(0.18)
GDP EU x CPBP	0.011	0.058	(0.00)	-	-	-
Unemp. EU x BDSF	-	-	-	0.002	0.188	(0.00)
VFTSE x CPBP	-	-	-	0.026	0.105	(0.00)
BIC_1	$\hat{\alpha}^{\sigma}$	$\hat{\alpha}^{\sigma}_{+}$	(p-value)	$\hat{\alpha}^{\gamma}$	$\hat{\alpha}^{\gamma}_{+}$	(p-value)
(Intercept)	10.001	10.007	(0.00)	-0.198	-0.223	(0.00)
IFRAUD	0.573	0.575	(0.00)	-0.020	-0.022	(0.21)
EFRAUD	0.59	0.600	(0.00)	-0.246	-0.160	(0.00)
EPWS	0.529	0.530	(0.00)	-0.108	-0.111	(0.00)
CPBP	1.102	1.101	(0.00)	-0.246	-0.340	(0.00)
BDSF	0.165	0.169	(0.00)	-0.080	-0.021	(0.28)
EDPM	0.722	0.727	(0.00)	-0.018	0.169	(0.00)
MIB	0.004	0.029	(0.07)	-	-	-
Unemp. IT	-	-	-	-0.031	-0.110	(0.00)
VFTSE	-	-	-	0.009	0.013	(0.45)
LR	-	-	-	-0.024	-0.081	(0.00)
GDP EU x CPBP	0.005	0.039	(0.01)	-	-	-
GDP IT x CPBP	-	-	-	0.016	0.045	(0.02)
Unemp. EU x BDSF	-	-	-	0.035	0.231	(0.00)
ST rate x BDSF	-	-	-	-0.005	-0.076	(0.00)
LOR IT x EFRAUD	-	-	-	0.026	0.128	(0.00)
LOR IT x EDPM	-	-	-	-0.018	-0.099	(0.00)
VFTSE x CPBP	-	-	-	0.057	0.150	(0.00)
TCR x EFRAUD	-	-	-	-0.061	-0.279	(0.00)
TCR x BDSF	-	-	-	0.006	-0.200	(0.00)
LR x EDPM	-	-	-	-0.001	-0.121	(0.00)

Table 3: Results of the regularized regressions for Model 3 using adLASSO penalties. Selection of the penalty parameters is performed over a grid using either BIC_2 as a criterion (top, $\boldsymbol{\nu}_2 = (0.0109; 0.0085)$) or BIC_1 (bottom, $\boldsymbol{\nu}_1 = (0.0140; 0.0051)$).

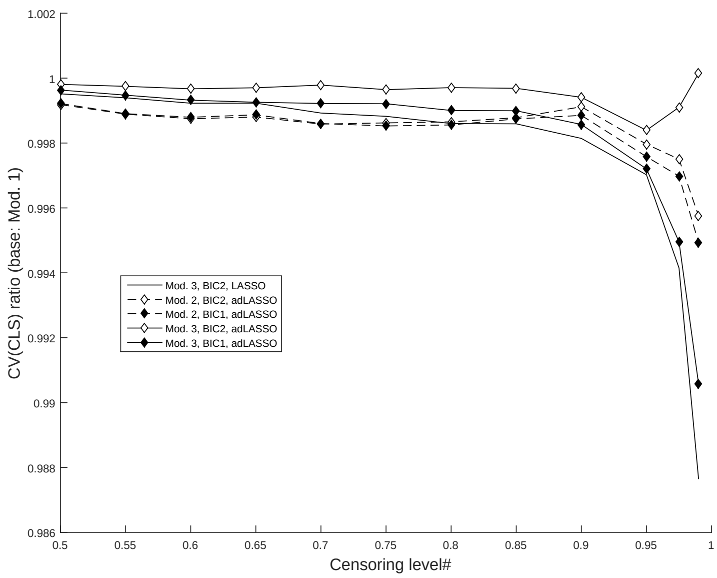

Figure 1: Cross-validated CLS for different values of κ, based on empirical quantiles. X-axis: quantile level used for the threshold. Y-axis: ratio between CV(CLS) of a given model and CV(CLS) of Model 1, such that values smaller than 1 are in favor of the alternative model. Solid: Model 3. Dashed: Model 2. $\diamondsuit$ (resp. $\blacklozenge$): selection based on BIC_2 (resp. BIC_1) and adLASSO.

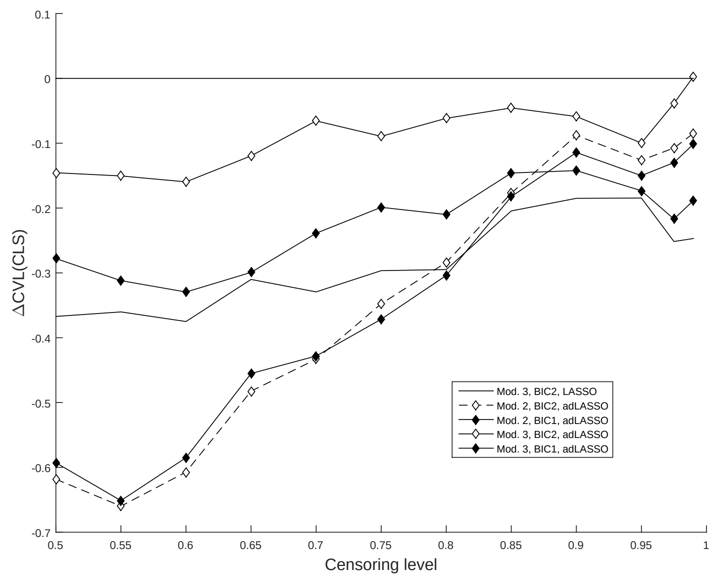

Figure 2: Same cross-validated CLS, expressed as a difference with respect to CV(CLS) of Model 1.

3 Sensitivity to threshold choice

In this section, we detailed the results obtained using either the 90% quantile ($\tau_1 = Q_{.9}(\mathbf{y}|ET)$) or the smallest loss size ($\tau_2 = \min\{\mathbf{y}|ET\}$) as thresholds (Table 4), to fit the full model. For τ_2 we observe a goodness-of-fit issue in the left tail of the residuals. We discard this threshold and continue the supplementary analysis only with τ_1. For comparison purposes, we also plot the residuals obtained with $\tau = Q_{.75}(\mathbf{y}|ET)$, Figure 3. Table 6 shows the results for Model 2 obtained with τ_1. We select essentially the same variables for γ: the Italian unemployment rate and the VIX. For σ we do not select any variable. As suggested by the simulation, when the sample size decreases we have difficulties in detecting the relevant covariates. Using BIC_1, we still did not select any variables for σ, but for γ we find the deposit growth and TR log-returns to be significant predictors. Looking at a model with interactions, we observe similarities with the solutions obtained for $Q_{.75}(\mathbf{y}|ET)$: we also select the VIX for γ and the interaction DGR x EFRAUD but for σ this time. We also select 2 others interactions with EFRAUD (with GDP and the short term rates), whereas no main effects for σ nor interaction effects for γ have been selected. Using BIC_1, we select in addition interactions of EU GDP and ST rates with EFRAUD, similarly to what we observe using $Q_{.75}(\mathbf{y}|ET)$. We can make the same observations for the main effect of the following variables: the Italian unemployment rate, DGR x IFRAUD and MIB x EPWS. Additional interactions with the S&P, the MIB and the TRSI are also selected (for EPWS, EDPM and EFRAUD), suggesting an association between financial markets and extremely large severities.

Unpen.	$\tau_1 = Q_{.9}(\mathbf{y}\|ET)$				$\tau_2 = \min\{\mathbf{y}\|ET\}$			
Covariate	$\hat{\alpha}^{\sigma}$	(p-value)	$\hat{\alpha}^{\gamma}$	(p-value)	$\hat{\alpha}^{\sigma}$	(p-value)	$\hat{\alpha}^{\gamma}$	(p-value)
(Intercept)	10.735	(0.00)	-0.214	(0.00)	8.532	(0.00)	-0.044	(0.00)
IFRAUD	0.631	(0.00)	-0.098	(0.00)	0.301	(0.00)	0.116	(0.00)
EFRAUD	0.471	(0.00)	-0.352	(0.00)	0.167	(0.00)	0.110	(0.00)
EPWS	0.515	(0.00)	-0.174	(0.00)	0.275	(0.00)	0.072	(0.00)
CPBP	1.014	(0.00)	-0.263	(0.00)	0.768	(0.00)	0.112	(0.00)
BDSF	0.242	(0.00)	-0.188	(0.00)	0.075	(0.00)	0.028	(0.00)
EDPM	0.860	(0.00)	-0.175	(0.00)	0.332	(0.00)	0.168	(0.00)
Unemp. EU	0.490	(0.00)	-0.202	(0.00)	-0.077	(0.00)	0.422	(0.00)
Unemp. IT	-0.243	(0.00)	-0.024	(0.46)	-0.153	(0.00)	-0.150	(0.00)
GDP EU	-0.029	(0.25)	-0.105	(0.00)	-0.023	(0.00)	0.031	(0.00)
GDP IT	-0.014	(0.58)	0.096	(0.00)	0.011	(0.18)	-0.001	(0.89)
HPI	-0.012	(0.63)	0.187	(0.00)	0.105	(0.00)	-0.006	(0.47)
M1	0.036	(0.16)	-0.035	(0.21)	0.027	(0.00)	-0.078	(0.00)
LT rates	-0.049	(0.06)	0.083	(0.00)	0.018	(0.04)	-0.022	(0.01)
ST rates	0.186	(0.00)	-0.171	(0.00)	-0.214	(0.00)	0.128	(0.00)
Stock returns	-0.240	(0.00)	0.047	(0.08)	-0.049	(0.00)	-0.043	(0.00)
S&P500	-0.086	(0.00)	-0.149	(0.00)	0.012	(0.15)	-0.033	(0.00)
TRSI	-0.036	(0.18)	0.297	(0.00)	-0.126	(0.00)	0.104	(0.00)
MIB	0.095	(0.00)	0.130	(0.00)	0.073	(0.00)	0.006	(0.46)
VIX	0.043	(0.10)	0.180	(0.00)	0.099	(0.00)	0.053	(0.00)
VFTSE	-0.335	(0.00)	0.239	(0.00)	-0.248	(0.00)	-0.008	(0.33)
LOR EU	0.282	(0.00)	0.064	(0.03)	0.086	(0.00)	0.150	(0.00)
LOR IT	-0.343	(0.00)	-0.034	(0.25)	-0.054	(0.00)	-0.013	(0.12)
PRF	-0.059	(0.02)	0.105	(0.00)	-0.028	(0.00)	0.022	(0.01)
DGR	-0.006	(0.83)	0.076	(0.01)	-0.016	(0.05)	0.021	(0.02)
TCR	-0.282	(0.00)	0.214	(0.00)	-0.124	(0.00)	0.033	(0.00)
LR	-0.131	(0.00)	0.047	(0.12)	-0.011	(0.20)	-0.024	(0.01)

Table 4: Results of the uneregularized regressions, using the event type and the economic covariates, with τ_1 (left) and τ_2 (right). P-values are in parentheses.

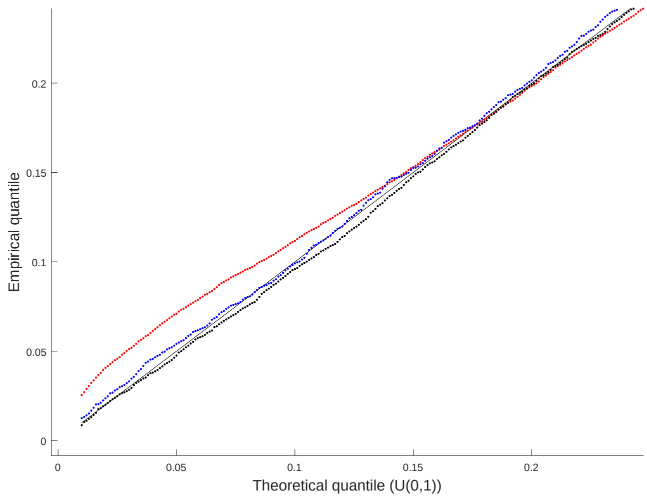

Figure 3: Dotted: (Zoomed) QQ-plots of the (pseudo-)residuals, obtained from the probability integral transform, for unregularized Model 2, using τ_1 (blue), τ_2 (red) and τ (black). Black solid: theoretical quantiles of U(0,1) distribution.

Model 2, LASSO						
BIC_2	$\hat{\alpha}^{\sigma}$	$\hat{\alpha}^{\sigma}_{+}$	(p-value)	$\hat{\alpha}^{\gamma}$	$\hat{\alpha}^{\gamma}_{+}$	(p-value)
(Intercept)	10.723	10.725	(0.00)	-0.166	-0.173	(0.00)
IFRAUD	0.621	0.621	(0.00)	-0.083	-0.084	(0.00)
EFRAUD	0.451	0.450	(0.00)	-0.335	-0.336	(0.00)
EPWS	0.500	0.500	(0.00)	-0.161	-0.158	(0.00)
CPBP	0.975	0.975	(0.00)	-0.216	-0.217	(0.00)
BDSF	0.223	0.223	(0.00)	-0.143	-0.142	(0.00)
EDPM	0.822	0.823	(0.00)	-0.137	-0.136	(0.00)
Unemp. IT	-	-	-	-0.020	-0.065	(0.04)
VIX	-	-	-	0.069	0.102	(0.00)
BIC_1	$\hat{\alpha}^{\sigma}$	$\hat{\alpha}^{\sigma}_{+}$	(p-value)	$\hat{\alpha}^{\gamma}$	$\hat{\alpha}^{\gamma}_{+}$	(p-value)
(Intercept)	10.723	10.73	(0.00)	-0.168	-0.186	(0.00)
IFRAUD	0.622	0.621	(0.00)	-0.083	-0.085	(0.00)
EFRAUD	0.452	0.451	(0.00)	-0.335	-0.336	(0.00)
EPWS	0.501	0.501	(0.00)	-0.160	-0.158	(0.00)
CPBP	0.976	0.976	(0.00)	-0.217	-0.222	(0.00)
BDSF	0.223	0.223	(0.00)	-0.142	-0.141	(0.00)
EDPM	0.824	0.825	(0.00)	-0.137	-0.136	(0.00)
Unemp. IT	-	-	–	-0.028	-0.064	(0.04)
TRSI	-	-	–	0.017	0.137	(0.00)
VIX	-	-	–	0.086	0.206	(0.00)
DGR	-	-	–	0.002	0.057	(0.04)

Table 5: Results of the regularized regressions for Model 2 and τ_1, using LASSO penalties. Selection of the penalty parameters is performed over a grid, using either BIC_2 (top) or BIC_1 (bottom). $\boldsymbol{\nu}_2 = (0.0125; 0.013)$ and $\boldsymbol{\nu}_1 = (0.15; 0.011)$.

Model 3, LASSO						
BIC_2	$\hat{\alpha}^{\sigma}$	$\hat{\alpha}^{\sigma}_{+}$	(p-value)	$\hat{\alpha}^{\gamma}$	$\hat{\alpha}^{\gamma}_{+}$	(p-value)
(Intercept)	10.723	10.726	(0.00)	-0.166	-0.175	(0.00)
IFRAUD	0.622	0.625	(0.00)	-0.083	-0.084	(0.00)
EFRAUD	0.427	0.377	(0.00)	-0.341	-0.351	(0.00)
EPWS	0.500	0.505	(0.00)	-0.162	-0.159	(0.00)
CPBP	0.973	0.984	(0.00)	-0.214	-0.220	(0.00)
BDSF	0.223	0.225	(0.00)	-0.143	-0.144	(0.00)
EDPM	0.821	0.833	(0.00)	-0.136	-0.143	(0.00)
VIX	-	-	-	0.052	0.111	(0.00)
S&P x EFRAUD	-0.005	-0.022	(0.32)	-	-	-
DGR x EFRAUD	0.008	0.029	(0.20)	-	-	-
ST rates x EFRAUD	0.030	0.103	(0.00)	-	-	-
BIC_1	$\hat{\alpha}^{\sigma}$	$\hat{\alpha}^{\sigma}_{+}$	(p-value)	$\hat{\alpha}^{\gamma}$	$\hat{\alpha}^{\gamma}_{+}$	(p-value)
(Intercept)	10.723	10.734	(0.00)	-0.172	-0.206	(0.00)
IFRAUD	0.622	0.625	(0.00)	-0.084	-0.111	(0.00)
EFRAUD	0.434	0.385	(0.00)	-0.350	-0.385	(0.00)
EPWS	0.501	0.512	(0.00)	-0.168	-0.214	(0.00)
CPBP	0.975	0.980	(0.00)	-0.216	-0.222	(0.00)
BDSF	0.223	0.225	(0.00)	-0.143	-0.144	(0.00)
EDPM	0.823	0.829	(0.00)	-0.137	-0.143	(0.00)
Unemp. IT	-	-	-	-0.018	-0.062	(0.05)
VIX	-	-	-	0.073	0.130	(0.00)
EU GDP x EFRAUD	-	-	-	-0.001	-0.018	(0.52)
ST rates x EFRAUD	0.024	0.102	(0.00)	-	-	
DGR x IFRAUD	-	-	-	0.001	0.045	(0.08)
DGR x EFRAUD	0.001	0.008	(0.73)	0.024	0.077	(0.02)
S&P x EFRAUD	-0.002	-0.014	(0.52)	-0.001	-0.017	(0.66)
TRSI x EPWS	-	-	-	0.002	0.043	(0.26)
MIB x EPWS	-	-	-	0.025	0.095	(0.02)
MIB x EDPM	-	-	-	0.008	0.071	(0.01)

Table 6: Results of the regularized regressions for Model 3 and τ_1, using LASSO penalties. Selection of the penalty parameters is performed over a grid, using either BIC_2 (top) or BIC_1 (bottom). $\boldsymbol{\nu}_2 = (0.156; 0.0189)$ and $\boldsymbol{\nu}_1 = (0.0168; 0.0131)$.

4 Quantile 95% of the severity

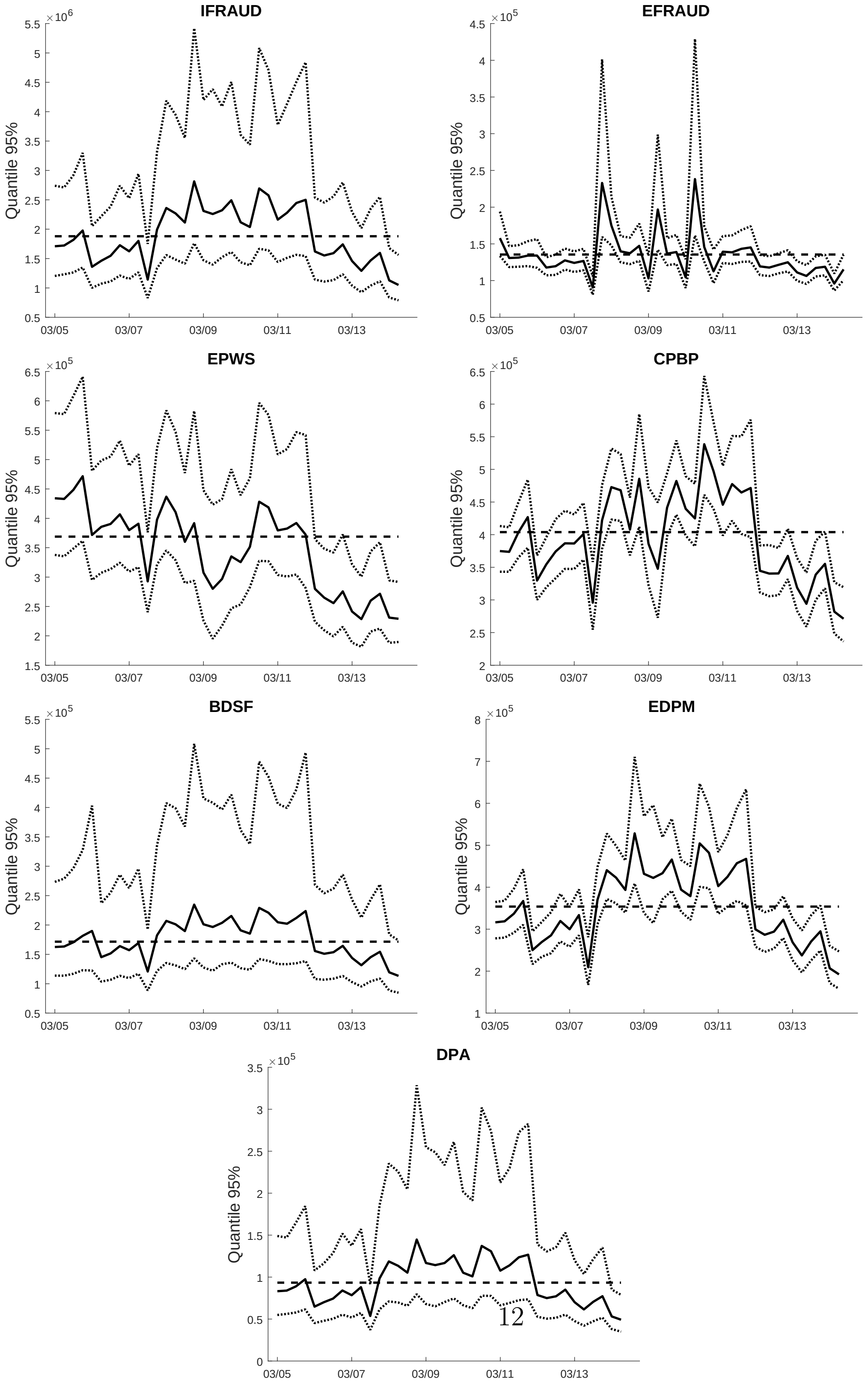

Figure 4: Solid: estimated quantile at level 95%, over time for Model 3 (penalized estimate based on BIC_2 with re-estimation step). Dotted: 95% confidence interval obtained with parametric bootstrap ($B = 5,000$). Dashed: estimated quantile for Model 1.