# Spatio-Temporal Expectile Regression Models
Supplementary Material

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## 1 Remarks

In this supplementary material plots that did not fit into the paper are displayed. These are the results of the simulation study for more asymmetry parameters and also for two numbers of observations (5000 as in the paper and 2000), the results of the application for more asymmetry parameters and the daily effect for four German cities which are mentioned in the paper. As in the paper it starts with the simulation study and continues with the application.

For explanations of the other supplementary material, i.e. the code and the data, please see the `README_SpatTemp.pdf` file.

## 2 Simulation Study

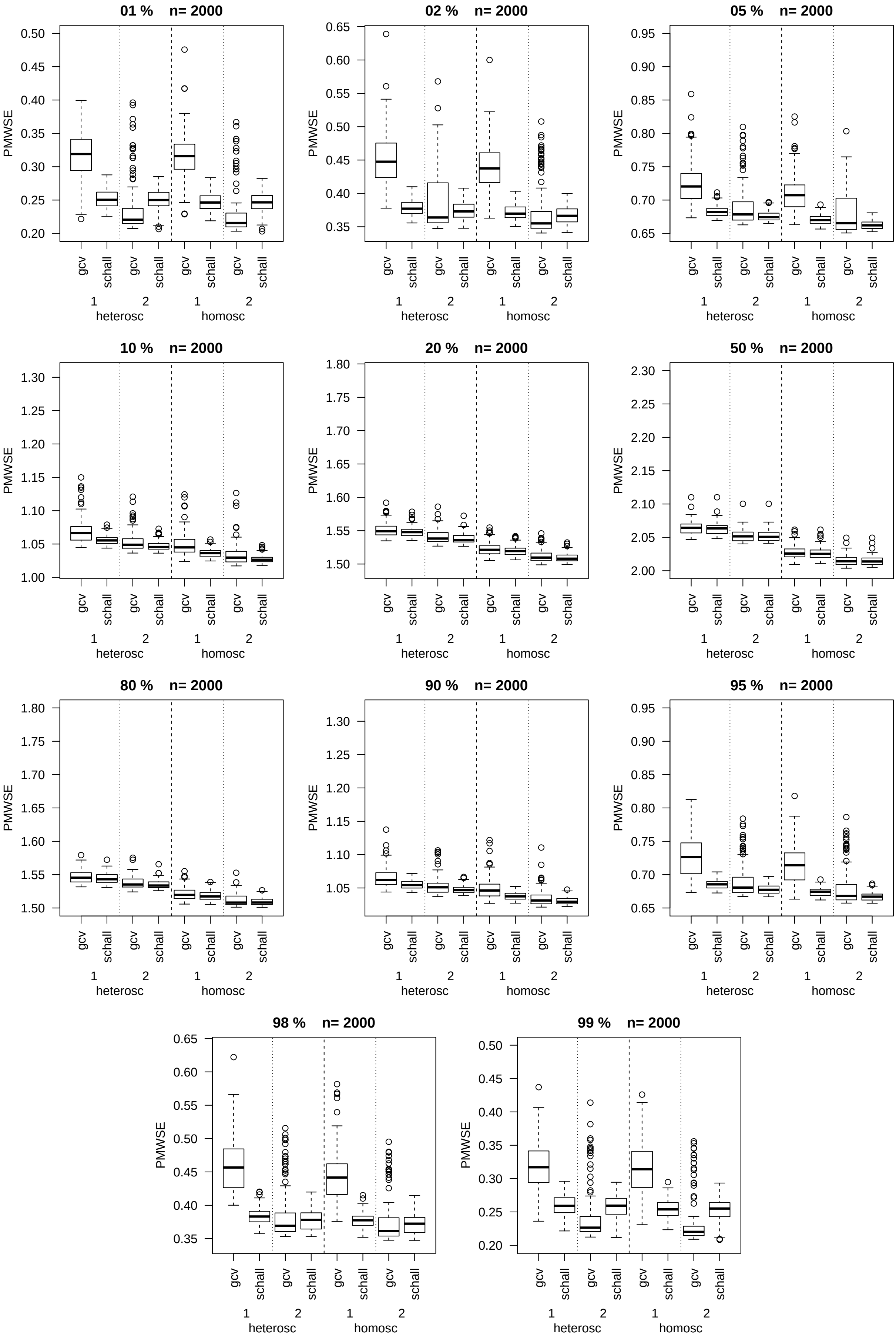


Figure 1: PMWSE of the simulation study for n=2000. On the left side of each plot the models with heteroscedastic errors are displayed and homoscedastic errors are on the right. For each data setting (1 or 2) the smoothing parameter selections are placed next to each other. The ranges of the y-axis are fixed to 0.30.

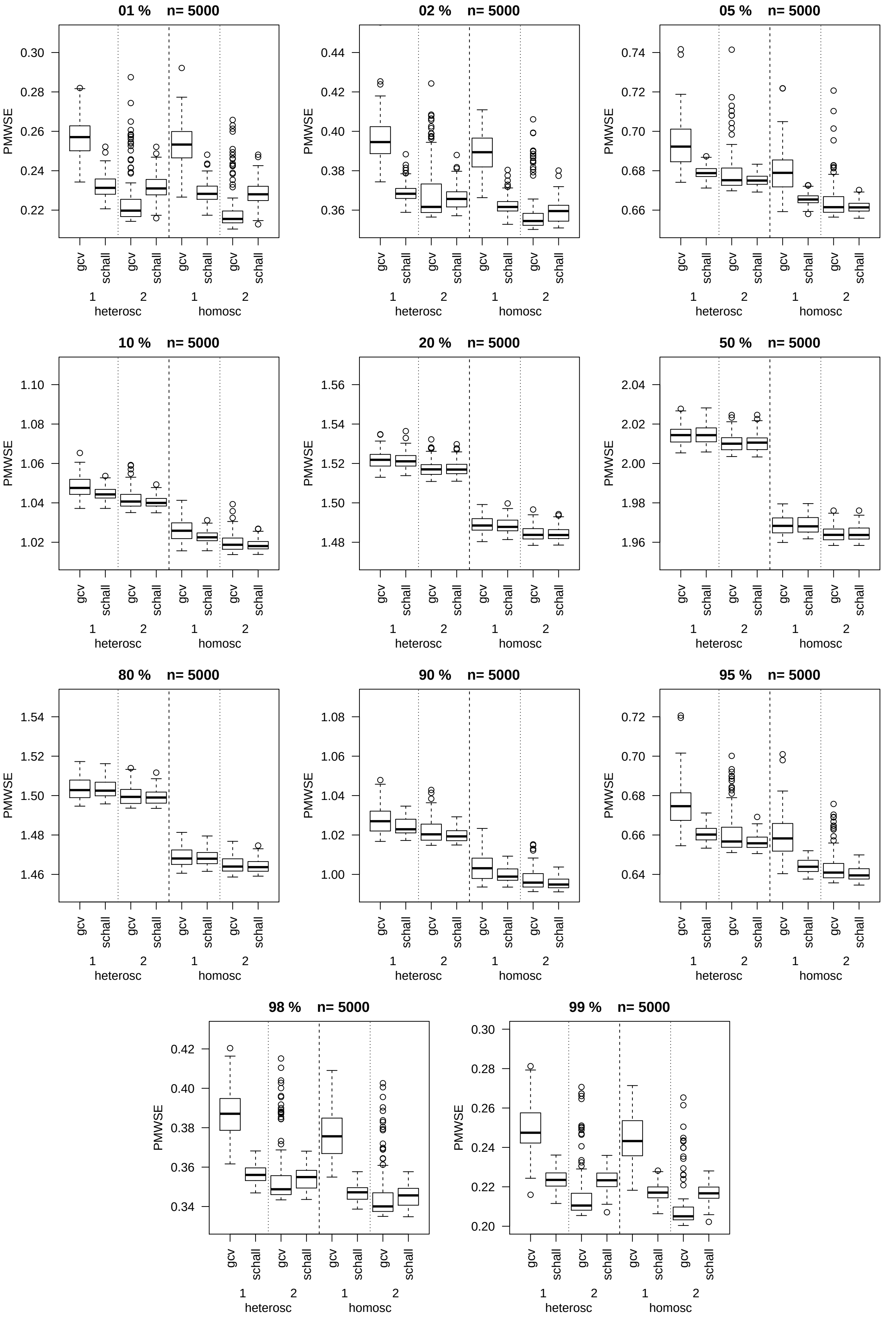


Figure 2: PMWSE of the simulation study for n=5000. On the left side of each plot the models with heteroscedastic errors are displayed and homoscedastic errors are on the right. For each data setting (1 or 2) the smoothing parameter selections are placed next to each other. The ranges of the y-axis are fixed to 0.10.

# 3 Application

## 3.1 Temporal Trend per City

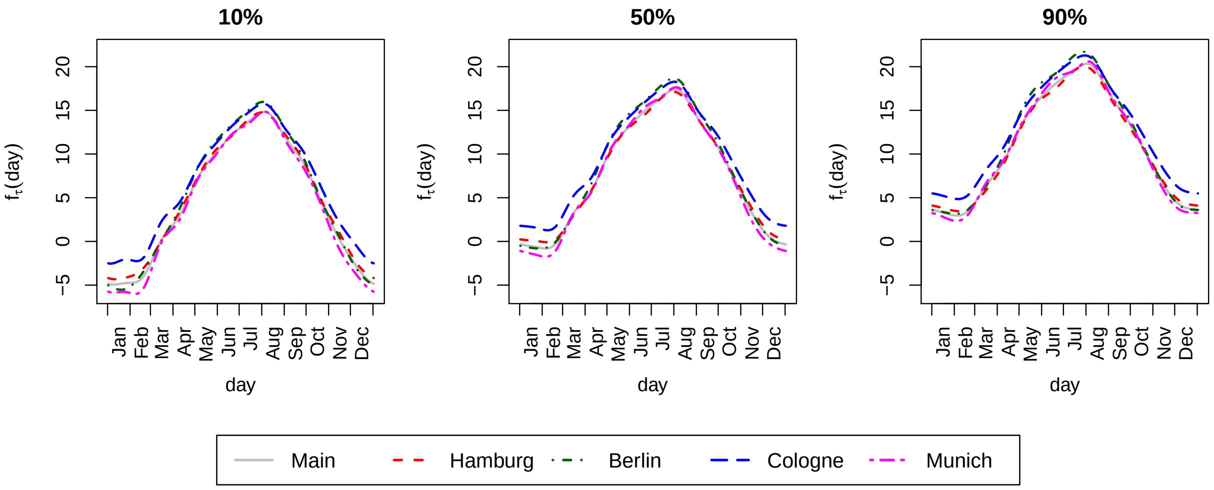


Figure 3: Temporal trend per city of the model based on trivariate splines.

## 3.2 Application for more asymmetry parameters

### 3.2.1 Model based on trivariate splines

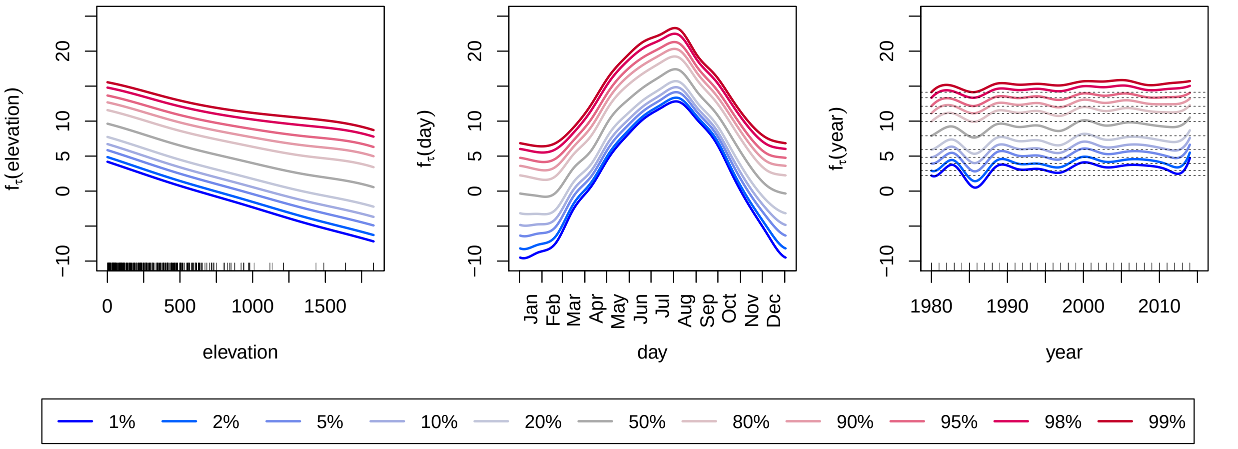


Figure 4: Main effects for elevation, day and year (including intercept). Based on the model using trivariate splines.

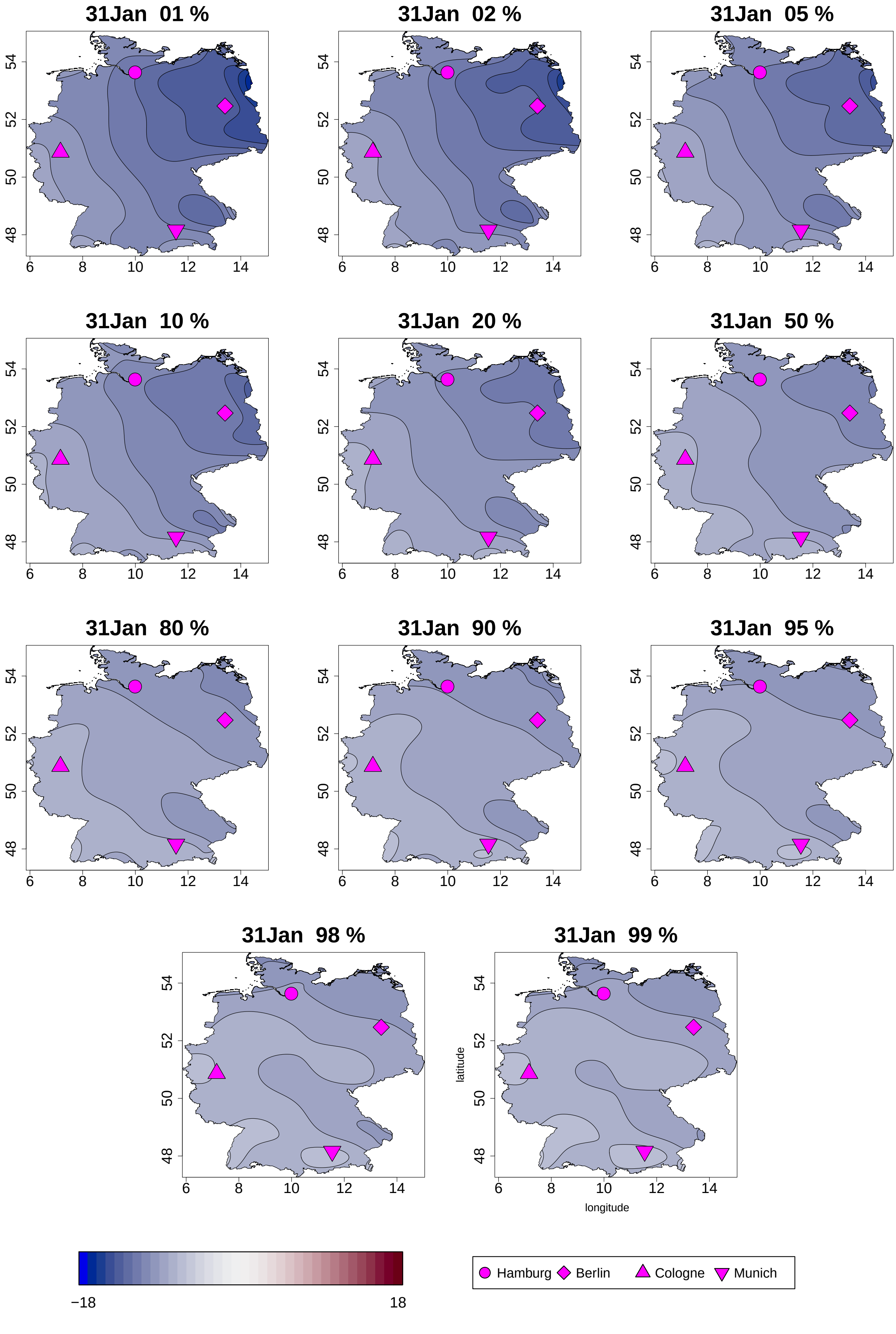


Figure 5: Spatial Effect for the 31st of January for several asymmetry parameters. The predictions are without the intercept! The range is here between $-18$ and 18 and not $-15$ to 15 as in the paper, but the steps are still $1°C$. Based on the model using trivariate splines.

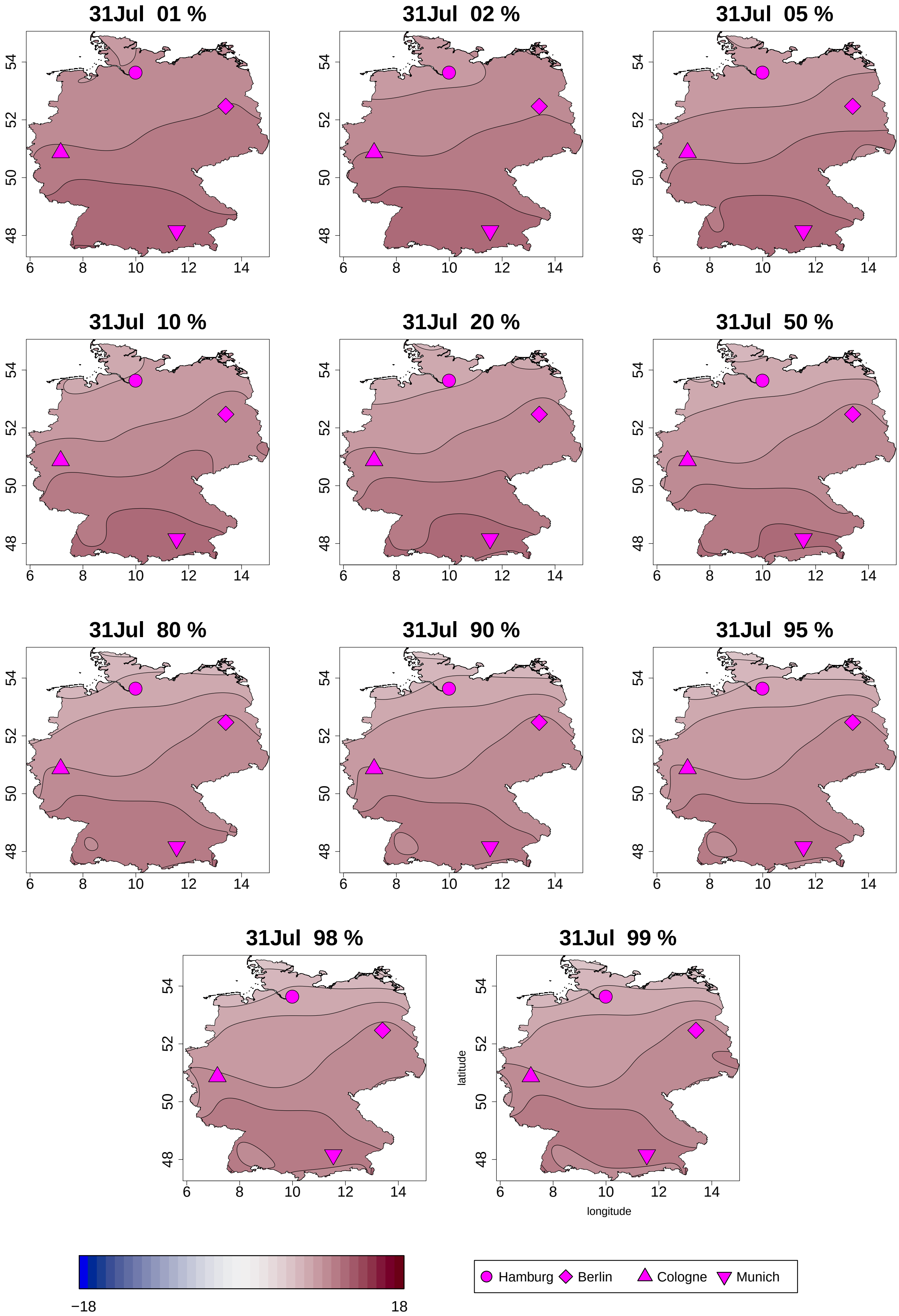


Figure 6: Spatial Effect for the 31st of July for several asymmetry parameters. The predictions are without the intercept! The range is here between −18 and 18 and not −15 to 15 as in the paper, but the steps are still $1°C$. Based on the model using trivariate splines.

### 3.2.2 Model based on GMRF

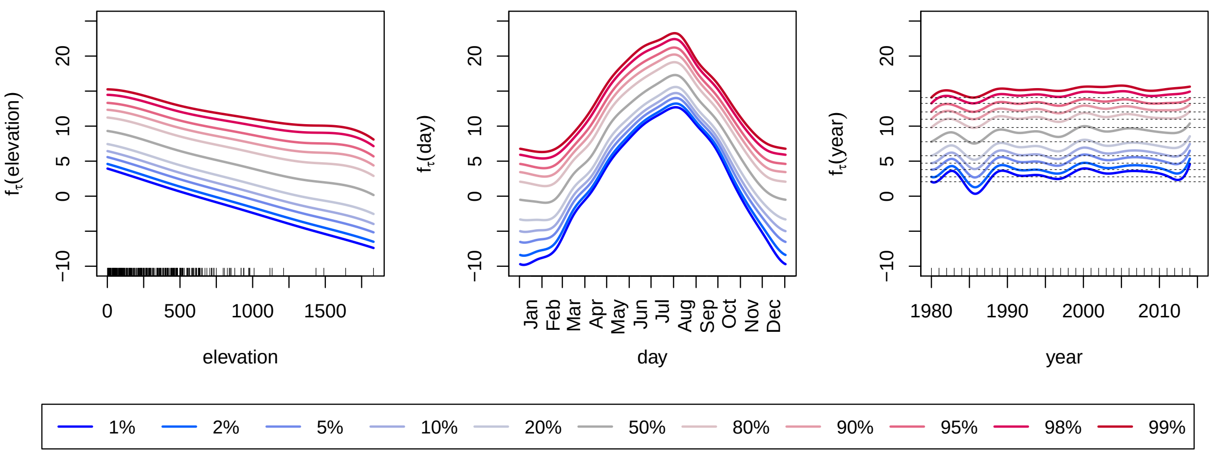


Figure 7: Main effects for elevation, day and year (including intercept). Based on the model using GMRF.

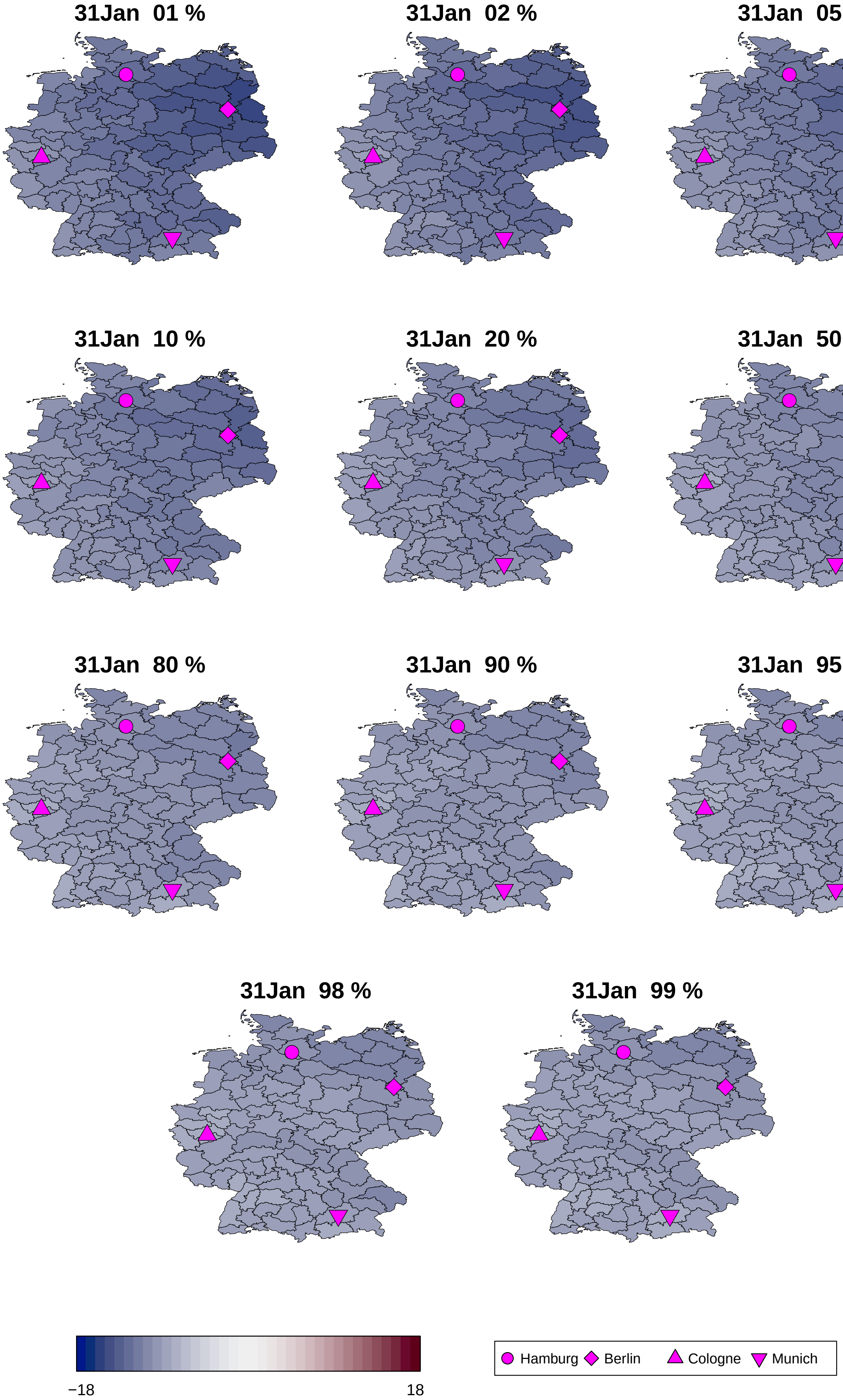


Figure 8: Spatial Effect for the 31st of January for several asymmetry parameters. The predictions are without the intercept! The range is here between $-18$ and 18 and not $-15$ to 15 as in the paper, but the steps are still $1°C$. Based on the model using GMRF.

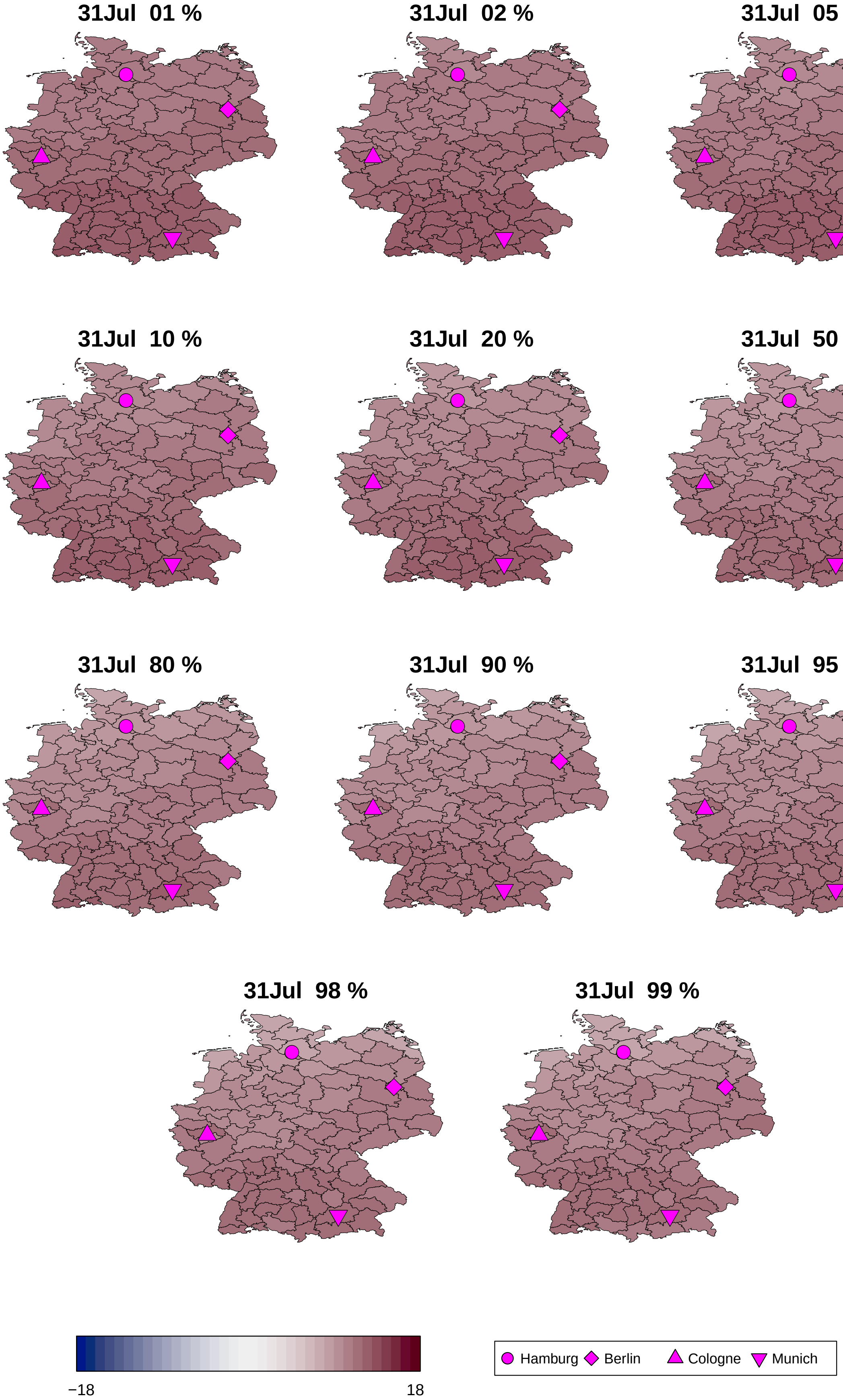


Figure 9: Spatial Effect for the 31st of July for several asymmetry parameters. The predictions are without the intercept! The range is here between $-18$ and 18 and not $-15$ to 15 as in the paper, but the steps are still $1°C$. Based on the model using GMRF.