Majority Vote on Educational Standards

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Abstract

The direct democratic choice of an examination standard, i.e., a performance level required to graduate, is evaluated against a utilitarian welfare function. It is shown that the median preferred standard is inefficiently low if the marginal cost of reaching a higher performance reacts more sensitively to ability for high than for low abilities, and if the right tail of the ability distribution is longer than the left tail. Moreover, a high number of agents who choose not to graduate may imply that the median preferred standard is inefficiently low even if these conditions fail.

Keywords: examination, school, drop-outs, democracy, median voter

JEL classification: I21, D72, I28

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1 Introduction

Education policy must trade off two conflicting goals. On the one hand, education aims at enhancing students' academic achievement. On the other hand, pressure to perform reduces students' well-being, as reported by the WHO (Inchley et al., 2020, p. 37-39). The present paper analyses how a democracy resolves this trade-off in relation to a key determinant of the incentives for learning and the associated psychological burden: the performance level required to graduate, called the examination standard.

Specifically, I characterise the majority vote on the standard and provide sufficient conditions implying that the standard chosen by a majority of voters is less demanding than would be efficient. These conditions mean that there are few highly able students who would easily reach higher standards, but these are outvoted by a majority of less able students who prefer a moderate standard. Thus, the theory proposed here explains why academic results barely improve in the log run (see OECD, 2023, p. 183), although raising educational achievements is a stated goal in most countries (see for example U.S. Department of Education, 2009; UK Government, 2018).

I develop a model where students of differing abilities decide how much effort to put into schooling. The effort determines the student's performance at the examination. If this performance meets or exceeds a level set by the school, called the examination standard, the student graduates, and otherwise she fails. Since employers observe only whether a student graduated or not but not the individual performance, the wage earned by graduates depends on the standard. Effort is costly, but more able students find it easier to comply with any given standard. For this reason, more able students prefer higher standards than students with lower abilities.

The standard is determined by a majority vote among agents, say the students' parents, who care for the interest of students. Although in reality, of course, examination standards are not literally voted upon in referenda, this modelling takes its relevance from a view of democracy, inspired by Downs (1957), according to which parties and other political actors have strong incentives to satisfy the wishes of the majority. Hence, by abstracting from details of the political process, the model focusses on the preferences of the electorate, arguably the most important determinant of policy.

In the main results it is analysed whether, starting from the standard preferred by agents with median ability, a marginal increase in the standard improves a utilitarian welfare criterion. Such a result obtains if two kinds of conditions are satisfied. The first type of condition requires that the marginal cost of satisfying a higher standard decreases more steeply in ability when ability is above the median than when it is below the median. The second kind of condition requires that the distribution of abilities is spread out more widely at the high end than at the low end. Intuitively, there is a lot to gain by tougher standards if there are a few students with very high abilities, and if for those students it is easy to satisfy more demanding standards.

In the model students are allowed to avoid the effort necessary to satisfy a standard they find too tough by not graduating. The existence of such 'drop-outs' creates an independent force which pushes towards an inefficiently low standard. Drop-outs are not affected by a marginal increase in standard since they anyway do not bear effort costs. Consequently, the standard should rise if there are many drop-outs. In an example, I illustrate that this effect may cause the democratically chosen standard to be too low even when the general condition on effort cost is not met.

In the following Section 2 I place my contribution in the context of the literature. The paper then continues in Section 3 with a description of the economic model. The main results on the welfare implication of majority voting are presented in Section 4, and Section 5 discusses the role of drop-outs. The final Section 6 offers some policy conclusions. Proofs and some methodological considerations are relegated to the supplementary material.

2 Literature Review

This paper is placed at the intersection of two strands of literature, the political economics of education and the analysis of examination standards. Research in the first strand is mainly concerned with the size of the budget available for public education or education subsidies, and the conflict of interest between different income classes when it comes to vote on the required taxes. As exemplified by Glomm and Ravikumar (1992), Haupt (2012), and Correa et al. (2020), the crucial feature in this type of analysis is the location of the decisive voter in the distribution of human capital or income. Major themes in this line of research are the choice between education expenditures and redistributive spending such as income transfers (Poutvaara, 2011; Bassetti and Greco, 2022) or pensions (Poutvaara, 2006; Lancia and Russo, 2016), or the allocation of funds between primary and higher education (Naito and Nishida, 2017). Further studies emphasise that also those voters who do not directly benefit from public education may support spending because a better educated workforce enhances productivity and growth (Creedy and Francois, 1990; Galor and Moav, 2006; Balestrino et al., 2021). Consequently, a variety of winning coalitions may occur (Fernandez and Rogerson, 1995; De Fraja, 2001; Levy, 2005; De Donder and Martinez-Mora, 2017; Lasram and Laussel, 2019).

As this brief overview shows, the political economics of education has so far mostly ignored the choice of graduation standards. The issue closest to the one analysed in the present paper is voting on admission standards. In De Fraja (2001)'s model, tightening admission rules can improve equality of opportunity since it re-allocates university places from low-ability higher income students to poorer students with high ability. De Donder and Martinez-Mora (2017) extend this model by allowing for private tuition expenditures which help prepare children for the admission test, but do not raise productivity at the workplace. Admission tests, as studied in these contributions, and graduation standards, as studied in the present paper, differ in one crucial respect: standards govern the incentives to learn. Therefore, like an admission test, the standard selects graduates, but, in addition, it also affects the productivity of each graduate. Extending previous knowledge, the present paper analyses how the interaction of learning incentives and the costs and rewards of academic achievement determine the outcome of a vote on standards.

The seminal contributions to the second strand of literature are Costrell (1994) and Betts (1998). In this work, the standard determines students' academic effort, and the school chooses the standard by trading off a higher wage for graduates against a larger number of graduates.

In line with the predictions from this theory, Figlio and Lucas (2004) show empirically that high school students with tough grading teachers perform better than those who are taught by lenient graders. Similarly, Babcock (2010) finds that study time at college is lower in courses with better grades.

Subsequent research has emphasised the information content of grades and degrees. In the model by De Paola and Scoppa (2010), students exert more effort when facing a more informative evaluation system since wages react more strongly to grades. In contrast, Collins and Lundstedt (2024) find that more informative grading deteriorates academic outcomes in Swedish schools because realistic feedback reduces the self-belief of students.

Schools often have an incentive to inflate grades. In the signaling model presented by Chan et al. (2007) schools always award better grades than deserved to at least some students. Such pooling arises since giving good grades is a costless way of raising the number of students who obtain high wages. Similarly, in the models by Ostrovsky and Schwarz (2010), Popov and Bernhardt (2013), and Zubrickas (2015), reducing the information content of transcripts or inflating grades allows schools to promote the job prospects of good or mediocre students at the expense of the truly excellent ones. When grades do not fully reveal ability, other signals such as the school's reputation or quality (MacLeod and Urquiola, 2015; Boleslavsky and Cotton, 2015; Ehlers and Schwager, 2016) or the student's social origin (Schwager, 2012; Tampieri, 2020) become relevant.

Research has also addressed the psychological cost of testing and more demanding standards. Linder et al. (2023a) find that levels of depression and anxiety rose among girls after Sweden introduced grades in 6th instead of 8th grade. According to Quis (2018), a reform in some German states which shortened the duration of secondary schooling by one year induced higher stress levels and more mental health problems, presumably because the same material had to be mastered in less time. In Germany, some but not all states hold central exit exams, which arguably represent a more demanding standard than school-administered tests, and do so only in a few subjects. Exploiting this two-way variation, Jürges and Schneider (2010) show that students who take central examinations in mathematics perform better but at the same time enjoy mathematics less and are more likely to find it boring. Taking the difference between school grades and central exam results as a measure of grade inflation at the school level, Linder et al. (2023b) show that over-grading improves the mental health of female students.

Special attention has been paid to gender aspects of grading. The psychological effects discovered by Quis (2018), Linder et al. (2023a), and Linder et al. (2023b) are concentrated in female students. In Portugal (Angelo and Balcão Reis, 2021) and Italy (Di Liberto et al., 2022), teachers grade boys less favourably than girls, measured against performance in standardised national tests. Intriguingly, as shown by Mechtenberg (2009), such a gender bias in grading may distort the beliefs of female students about their own ability and hence lead to inefficient career choices.

Finally, the incentives exerted by grading standards can also explain peer effects, as shown empirically by Kiss (2018) and theoretically by Ehlers and Schwager (2020). This arises because schools set a higher standard in an academically stronger class, incentivising weaker students to perform better in such a class.

Only few contributions apply a political economy approach to the choice of graduation standard. In a short section on this issue Costrell (1994, p. 963-964) concludes that the democratically chosen standard is excessively tough, based on assumptions resembling the conditions which in the present model imply an inefficiently low standard. Both approaches differ in that Costrell's assumptions restrict the distribution of voters' preferred standards, whereas the conditions used here relate to primitives. More importantly, in Costrell (1994) voters, like schools, are only concerned with wages and educational outcomes but do not take students' effort into account. In the model presented here, such a disutility of learning is a major driver of voters' decisions, and consequently the chosen standard tends to be lower. In this sense, my model is tailored to parents who take care to protect their children from psychological cost of schooling, as documented by Jürges and Schneider (2010), Linder et al. (2023b), and Inchley et al. (2020). In contrast, Costrell (1994) rather features an aspiring type of parents who push their children to highest performance, disregarding students' disutility of learning.

In the model by Brunello and Rocco (2008), a profit maximising private school competes with a public school whose standard is set by majority vote. These authors show that two regimes can arise in equilibrium: Either the public school sets the most demanding and the private school sets the most lenient among all admissible standards, or vice versa. At least for parameters calibrated to the U.S. and Italy, these standards maximise utilitarian welfare. While this contribution shares some features with the present paper, both provide different insights. Brunello and Rocco (2008) focus upon the interaction between private and public schools, which is absent in my paper. Thus, my model is tailored towards school systems where a unique examination standard is centrally chosen and applied to all students. Moreover, I relate general properties of the effort cost function and the distribution of abilities to the efficiency of the chosen standard, where Brunello and Rocco (2008) fix a log-normal distribution and linear cost of effort. My results suggest that an interior, uniform standard chosen by the median voter is less likely to be efficient than a menu of two extreme standards parents may choose from.

While focussed upon the individual incentives of more informative grading, De Paola and Scoppa (2010, p. 207) mention that in their model, a majority of low ability students would vote for the less informative grading system. Unlike these authors I model the vote and the welfare assessment explicitly. This allows me to show that the interaction of the shapes of the marginal effort cost function and of the distribution of abilities are crucial for the outcome.

The impact of a higher college wage premium on university quality, which is equivalent to the standard, is in the focus of the contribution by Meier and Schiopu (2020). These authors show that, to accommodate higher enrollment, quality decreases both in a system where colleges are differentiated by quality, as in the United States, and in a system where a uniform quality is determined politically, as in most European countries. This work differs from the analysis presented here with respect to the political process underlying the choice of the uniform standard. While Meier and Schiopu (2020) use a probabilistic voting approach which is equivalent to maximising a welfare function, the discrepancy between the welfare maximising standard and the median's choice is at the center of the present analysis. This discrepancy provides a distinct mechanism by which quality decreases, which potentially adds to the impact of higher enrollment described by Meier and Schiopu (2020).

3 A Model of Graduation Standards

There is a continuum of agents with mass one. Agents have two roles in the model, as students and as voters. One can interpret agents literally as adult individuals who still are in education, for example at a university, at an age where they have the right to vote. More broadly, agents can be seen as representing families composed of children in education and parents who use their right to vote to promote the interests of their children.

Agents are characterised by their ability $a \in A := [a_o, a_1)$, where $a_o \geq 0$ and a_1 may be infinite. Abilities are distributed according to the c.d.f. $F : A \to [0,1]$ which is continuous and strictly increasing on the support A. Thus, the density is strictly positive for $a \in A$. The mean and median abilities are denoted by $\overline{a} = \int_A a \, dF(a)$ and $a_m = F^{-1}(1/2)$.

To succeed at school, agents must exert effort denoted by $e \geq 0$. Effort can be interpreted as any disutility created by learning. This encompasses the opportunity cost of the time spent at school or studying, where students otherwise could go out with friends, play video games, practice sport, or pursue other extracurricular activities. Moreover, effort also relates to psychological cost of studying hard, such as losing the joy of learning, being stressed, arguing with parents over homework, or impaired mental health.

An agent with ability a who provides effort e incurs cost $c(e,a)$. The cost function $c : \mathbb{R}_{\geq 0} \times A \to \mathbb{R}_{\geq 0}$ is assumed to be three times continuously differentiable. I denote derivatives by subscripts; for example, $c_e(e,a)$ is the partial derivative of cost with respect to effort.

Assumption 1 $c(0,a) = 0$ *and* $\lim_{e\to\infty}[c(e,a)/e] > 1$ *for* $a \in A$.

Assumption 2 $c_e(0,a) = 0$ *for* $a \in A$, $c_e(e,a) > 0$ *and* $c_a(e,a) < 0$ *for* $(e,a) \in \mathbb{R}_{>0} \times A$

Assumption 3 $c_{ee}(e,a) > 0$, $c_{aa}(e,a) > 0$, *and* $c_{ea}(e,a) < 0$ *for* $(e,a) \in \mathbb{R}_{>0} \times A$

According to Assumption 1, a student who does not exert any effort does not incur any cost, and for increasing effort, the cost eventually exceeds the effort. Assumption 2 says that cost increases in effort, starting with a marginal cost of zero, but decreases in ability. Assumption 3 states that the marginal cost of effort is strictly increasing, that the cost-saving effect of ability becomes weaker (in absolute terms) as ability increases, and that higher ability decreases the marginal cost of effort.

The standard $s \in \mathbb{R}_{\geq 0}$ defines the performance level required to pass the examination. Performance is entirely determined by, and measured in the same units as, effort, subsuming the influence of ability in the cost function $c(e,a)$. Students who exert effort $e \geq s$ graduate, while those with $e < s$ fail and will be referred to as drop-outs.

After leaving school, agents will be employed by firms which operate a constant returns to scale technology transforming one efficiency unit of labour into one unit of a numéraire output. The amount of efficiency units supplied by a worker is given by her examination performance, and hence by the effort level e deployed at school. Firms cannot observe the examination performance of an individual worker but know whether she graduated or not. Therefore, all graduates will obtain the same wage w_s, and all drop-outs will receive the same wage w_o. In a competitive equilibrium on the labour market, the graduate wage w_s (the drop-out wage w_o) must be equal to the expected productivity of all agents who exert effort $e \geq s$ ($e < s$). Ownership of firms is sufficiently widely distributed among the agents that price-taking behaviour is justified, but otherwise need not be specified. The reason is that, because of constant returns to scale, firms earn zero profits whatever the standard or the behaviour of students, so that firm ownership does not change the stakes any individual has in the choice of standards.

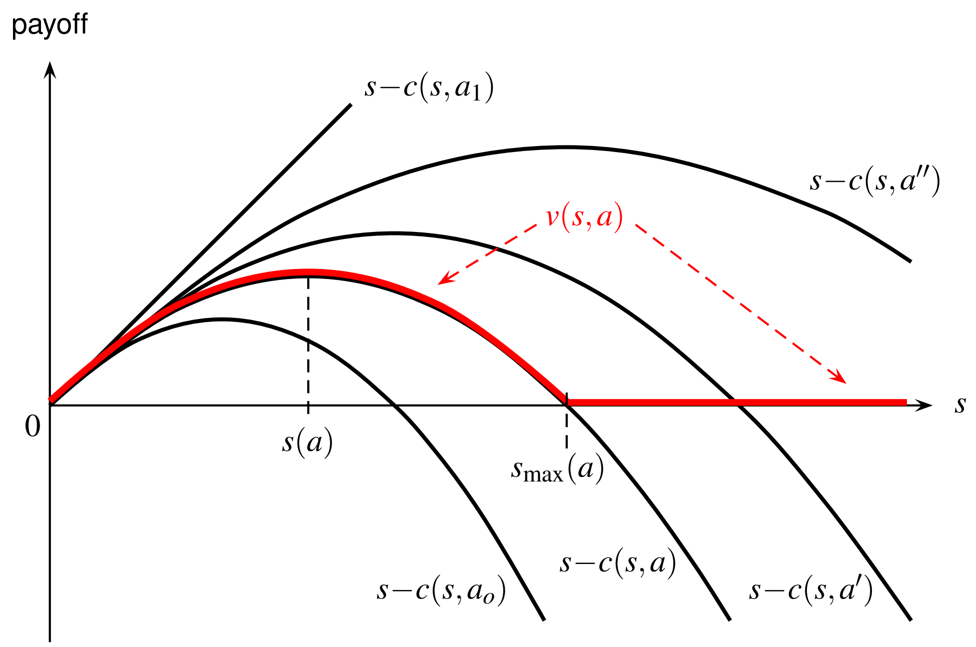
For given standard s, a student of ability $a \in A$ chooses effort so as to maximise the expected wage net of effort cost. Conditional on choosing an effort $e \geq s$ sufficient to graduate this payoff is $w_s - c(e,a)$. Since the wage does not depend on effort as long as the constraint $e \geq s$ is met, from $c_e > 0$ the minimal effort $e = s$ dominates all effort levels $e > s$. In the same way, conditional on not graduating ($e < s$), the payoff is $w_o - c(e,a)$ which is maximised by $e = 0$. Thus, students either just meet the standard and graduate, or they do not put in any effort at school and fail. Observing $c(0,a) = 0$ from Assumption 1, one sees that graduation (dropping out) is optimal if $w_s - c(s,a) \geq (<)\, w_o$.

With this behaviour, equilibrium wages will be $w_s = s$ and $w_o = 0$. Thus, in equilibrium graduation is optimal if

$$s - c(s,a) \geq 0\,. \tag{1}$$

Figure 1 shows the payoff from graduating on the l.h.s. of (1) as a function of the standard s for several levels of ability. From Assumption 1, this payoff is zero at $s = 0$ and eventually becomes negative for high enough s. Moreover, from Assumption 2, the payoff's slope $1 - c_e(s,a)$ is positive at $s = 0$ implying that for all $a \in A$, at some standard, graduation is worthwhile and a positive payoff can be reached.

Figure 1: The payoff from graduating and indirect utility



The curves show the payoff from graduating $s - c(s,a)$ as a function of the standard s, for agents with varying abilities $a_o < a < a' < a'' < a_1$. The indirect utility $v(s,a)$ of an agent with ability a is displayed in bold red. For this agent, the optimal standard is $s(a)$, and the highest standard such that she will graduate is $s_{\max}(a)$.

Assumptions 1 to 3 imply that for each $a \in A$ there is a unique positive standard $s_{\max}(a)$, given by the solution to (1) as an equality, which yields a payoff of zero. As is apparent from Figure 1, an agent with ability a will graduate if $s \leq s_{\max}(a)$ and drop out if $s > s_{\max}(a)$. Thus, $s_{\max}(a)$ is the maximal standard which an agent of ability a is willing to satisfy.

Inverting the relationship $s_{\max}(a)$, one can express the decision to graduate or not by defining a minimal ability $a_{\min}(s)$ which an agent must have to be willing to satisfy a given standard s. This level is called the graduation threshold for standard s. If all agents graduate or drop out under a standard s, I define the graduation threshold to be a_o or a_1, respectively:

Definition 1 *For all $s \in \mathbb{R}_{\geq 0}$, the graduation threshold is*

$$a_{\min}(s) = \begin{cases} a_o & \text{if} \quad s - c(s,a) > 0 \quad \text{for all } a \in A \\ \tilde{a} & \text{if} \quad s - c(s,\tilde{a}) = 0 \quad \text{for some } \tilde{a} \in A \\ a_1 & \text{if} \quad s - c(s,a) < 0 \quad \text{for all } a \in A\,. \end{cases}$$

Notice that from $c_a < 0$ in Assumption 2, $\tilde{a}$ in the second line of Definition 1 must be unique if it exists and hence $a_{\min}(s)$ is well defined. Moreover, all agents with $a < a_{\min}(s)$ will fail and all agents with $a \geq a_{\min}(s)$ graduate. Finally, differentiating $s - c(s,a) = 0$ shows that

$$\frac{da_{\min}(s)}{ds} = \frac{1 - c_e(s, a_{\min}(s))}{c_a(s, a_{\min}(s))} > 0\,,$$

where the inequality follows on $c_a < 0$ and the fact that at $a_{\min}(s)$, the payoff from graduating must be decreasing in the standard. Therefore, the graduation threshold is weakly increasing in the standard s, and strictly so if the threshold is in the interior of A.

Before turning to the analysis of the political equilibrium, I briefly discuss the way I model standards and the information they provide. The key assumption states that by applying the standard, the school maps the student's performance, which is measured continuously, into a binary signal observed by the firm. This signal can be interpreted more broadly than graduation or not, meaning for example a good vs. a mediocre grade, or graduating with vs. without distinction. Similarly, payoffs can be interpreted as any benefit students obtain from school which differs by graduation status or grade, such as the probability of being admitted to a prestigious further education institution, or the probability of obtaining a fellowship. Moreover, the analysis remains essentially unchanged if there are more than two values for the signal, for example in the form of four grades, as students will still put in just enough effort to make the desired grade, or fall back to the threshold for the next-lower grade.

To have a meaningful model of standards, however, it is essential that the signal available to firms stems from a coarser information partition than the underlying performance. To see why, consider that, instead, schools provide an unbiased and fully informative assessment of performance. In this case firms just pay each worker according to her individual performance and do not care whether this performance happens to meet the standard defined by the school. Grade point averages (GPAs) might provide such a signal. However, even if observed, firms may be reluctant to solely rely on GPAs because grades are noisy, such that small differences in GPA are not informative, or because going through transcripts is too time consuming. Moreover, schools, students, and employers apparently take degrees very seriously. In particular, obtaining a degree increases earnings discontinuously, an observation which has been termed the 'sheepskin effect' (Heckman et al., 2006, p. 330). This suggests that the information provided by a standard is relevant even if GPAs are available.

4 Voting and Welfare

From the optimal decision to graduate, one finds the indirect utility function $v(s,a) = \max\{0; s - c(s,a)\}$ of an agent $a \in A$. This function, which in Figure 1 is illustrated by a bold red line, relates the standards about which agents vote to the individual agent's utility, anticipating her own effort and graduation choices and the equilibrium wages ensuing from the chosen standard. From Assumption 3, the payoff from graduating is strictly concave in s. Hence, for each $a \in A$, there is a unique standard $s(a) = \text{argmax}_s\{v(s,a) | s \geq 0\} > 0$ which maximises the indirect utility of an agent with ability a (see Figure 1).

Increasing ability shifts the payoff from graduating upward, since Assumption 2 implies that cost decreases in ability. Moreover, from Assumption 3, the marginal cost of effort decreases when ability rises. As illustrated in Figure 1, this means that both the utility maximising standard $s(a)$ and the highest standard $s_{\max}(a)$ which an agent will satisfy strictly increase in ability a.

The indirect utility function represents preferences of students and voters. Interpreting voters as parents, I thus assume that parents take the disutility of learning experienced by their children into account when casting their vote on education policy. While this might appear implausible when the alternative to learning is playing video games, parents may well appreciate other time uses of their children such as sports or cultural activities. Moreover, and arguably more importantly, parents likely are concerned with stress or mental health issues attributed to pressure in school and try to influence policy towards reducing such psychological cost.

The standard is determined by the agents in a series of pairwise votes. The voting equilibrium is described by a Condorcet winner, that is, a standard which collects a majority of votes against any other standard. In the present context, the analysis of Condorcet winners is complicated by the opportunity to drop out. With this option, indirect utility $v(s,a)$, while strictly increasing in s for standards $s < s(a)$, is only weakly decreasing for standards $s > s(a)$. To account for this, a definition of single-peakedness which allows for flat parts of the indirect utility function, as in Persson and Tabellini (2002, Definition 2, p. 22), can be employed. Equivalently, I define a Condorcet winner by requiring only that the winning standard must be weakly but not necessarily strictly preferred to any other standard by a majority of agents:

Definition 2 *Standard $s \in \mathbb{R}_{\geq 0}$ is a Condorcet winner if for all standards $s' \in \mathbb{R}_{\geq 0}, s' \neq s$:*

$$\int_{\{a \in A | v(s,a) \geq v(s',a)\}} dF(a) > 1/2.$$

With this definition, a median voter result obtains:

Proposition 1 *The standard $s_m := s(a_m)$ preferred by agents with median ability is a Condorcet winner.*

Proof: See Appendix A.I in the supplementary material. ■

The median preferred standard s_m is not the unique Condorcet winner. A somewhat higher standard $\tilde{s} > s_m$ can be supported as a Condorcet winner as well if agents at the low end of the ability distribution drop out under both standards and, being indifferent, support the higher standard in a pairwise vote. (For an example, see Appendix A.II in the supplementary material.) Conversely, the median preferred standard is the unique Condorcet winner if almost all agents graduate under it, $a_{\min}(s_m) = a_o$. More generally, one can restore uniqueness by refining the concept of Condorcet winner in the spirit of trembling-hand perfection. If all agents have a small probability of graduating 'erroneously' the indifference of low ability agents is resolved in favour of the less demanding standard, and the median preferred standard is the only Condorcet winner robust to such perturbations. (Details are presented in Appendix A.III in the supplementary material.) For these reasons, the welfare analysis is focussed upon on the median preferred standard.

Welfare is defined by a utilitarian criterion, aggregating the indirect utility of all agents:

Definition 3 *For any standard s, welfare is*

$$W(s) = \int_{a_o}^{a_1} v(s,a)\, dF(a)\,.$$

For agents who graduate, utility is the wage earned net of effort cost, and for drop-outs, utility is zero. Therefore, welfare is given by $W(s) = \int_{a_{\min}(s)}^{a_1} \left[s - c(s,a)\right] dF(a)$. Differentiating this equation w.r.t. s and using Definition 1, one finds that an increase in the standard changes welfare by

$$W'(s) = \int_{a_{\min}(s)}^{a_1} \left[1 - c_e(s,a)\right] dF(a)\,. \tag{2}$$

To understand (2), notice that a change in the standard affects both the graduation threshold $a_{\min}(s)$ and the utilities $v(s,a)$. The first effect cancels, however, since the utility of an agent at the threshold is zero by definition. Since drop-outs anyway receive a utility of zero, the second effect is relevant only for those agents who will graduate under the original standard. For these individuals, raising the standard by one unit increases the wage by one unit, since wage and standard are normalised to be equal. On the other hand, in order to satisfy the higher standard, students have to incur additional effort cost so that, for an agent with ability a, the net gain from increasing the standard is $1 - c_e(s,a)$.

In the following I will examine under what conditions welfare will increase if the standard is raised above the standard chosen by the majority. That is, I provide sufficient conditions for $W'(s_m) > 0$. Only a local welfare analysis is offered since any second order conditions ensuring a global maximum will necessarily require assumptions on the shape of the density $F'(a)$, which are likely to be very strong or difficult to interpret.

The starting observation in this analysis is that, because s_m is optimal, agents with median ability are indifferent to an increase in standard, $1 - c_e(s_m, a_m) = 0$. Since marginal cost of effort is strictly decreasing in ability, agents with above-median ability will gain from an increase in standard, i.e., $1 - c_e(s_m, a) > 0$ for all $a > a_m$. Agents with below-median ability will lose, $1 - c_e(s_m, a_m) < 0$, as long as they still graduate. Therefore, the net welfare effect of an increase in the standard hinges on the relative sizes of aggregate gains and losses by high and low ability agents respectively. These aggregate amounts in turn are determined by three features: the shape of the marginal cost function c_e, the distribution function $F(\cdot)$, and the graduation threshold $a_{\min}(s_m)$. In this section, I provide two results highlighting the role of the first two features, whereas the importance of the graduation threshold is taken up in Section 5.

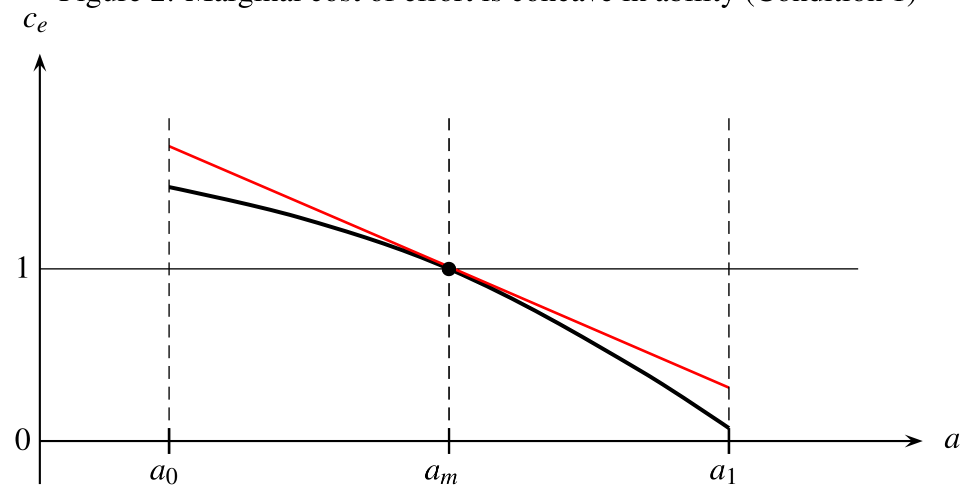
The properties of the cost and distribution functions used in these results are described by two pairs of conditions. The first pair are

Condition 1 $c_{eaa}(s_m, a) \leq 0$ *for* $a \in [a_o, a_1)$.

Condition 2 $\overline{a} \geq a_m$.

Condition 2 simply states that mean ability exceeds median ability. Condition 1, which is illustrated in Figure 2, requires that the marginal cost of effort c_e is a concave function of ability. In Figure 2, ability is depicted on the horizontal axis and marginal cost and benefits of an increase in standard are measured on the vertical axis. The marginal cost of effort evaluated at the median preferred standard, $c_e(s_m, a)$, decreases in ability according to Assumption 3, and cuts the marginal benefit of 1 at the median ability a_m. As illustrated in this figure, if the cost function satisfies Condition 1, the marginal cost curve should become steeper as ability increases. Thus, the effort-enhancing effect of ability increases in ability.

Figure 2: Marginal cost of effort is concave in ability (Condition 1)



The bold black curve shows the marginal cost of effort at the median preferred standard $c_e(s_m, a)$ as a function of ability. The horizontal line represents the marginal benefit of an increase in the standard. Condition 1 requires that the marginal cost stays below the tangent, painted red.

Alternatively, I consider the following pair of conditions:

Condition 3 *For all* $x \in (0, \min\{a_m - a_o; a_1 - a_m\}]$*:*

$$\frac{1}{2}c_e(s_m, a_m - x) + \frac{1}{2}c_e(s_m, a_m + x) \leq 1\,.$$

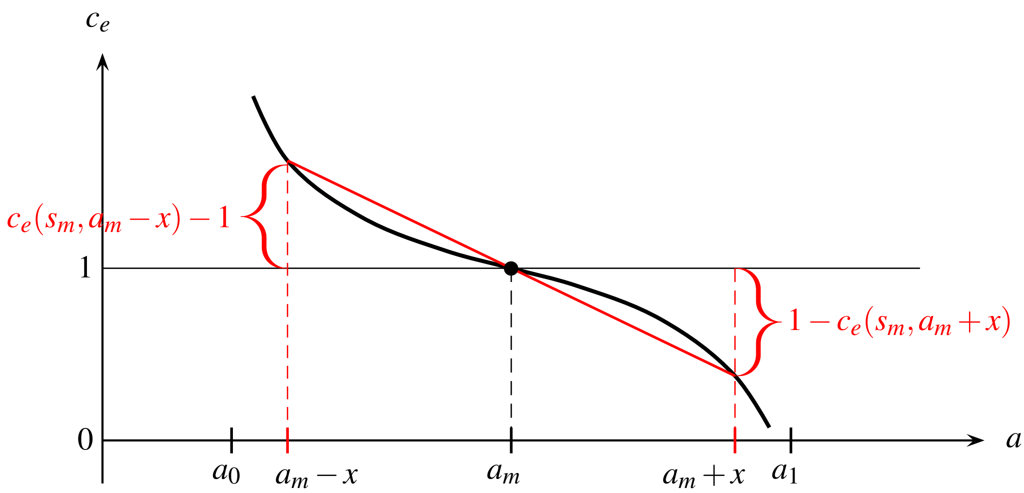
Condition 4 *For all* $x \in (0, \min\{a_m - a_o; a_1 - a_m\}]$*:*

$$\frac{1}{2} - F(a_m - x) \geq F(a_m + x) - \frac{1}{2}\,.$$

In Condition 3, two agents are considered whose abilities exceed and, respectively, fall short of the median ability by the same amount x. The condition requires that the average marginal cost of these two individuals does not exceed the marginal benefit. Thus, on average, these two agents gain from raising the standard. Figure 3 gives a geometric intuition for this property, which is based on splitting the graph of the marginal cost curve c_e in the two parts corresponding to the domains of below and above median abilities. Condition 3 requires that, when one of these parts is mirrored at the point $(a_m, 1)$, the image should be located below the other part.

Also Condition 4 (see Figure 4) starts from considering two ability levels which are located

Figure 3: Marginal effort cost at abilities symmetric to the median (Condition 3)



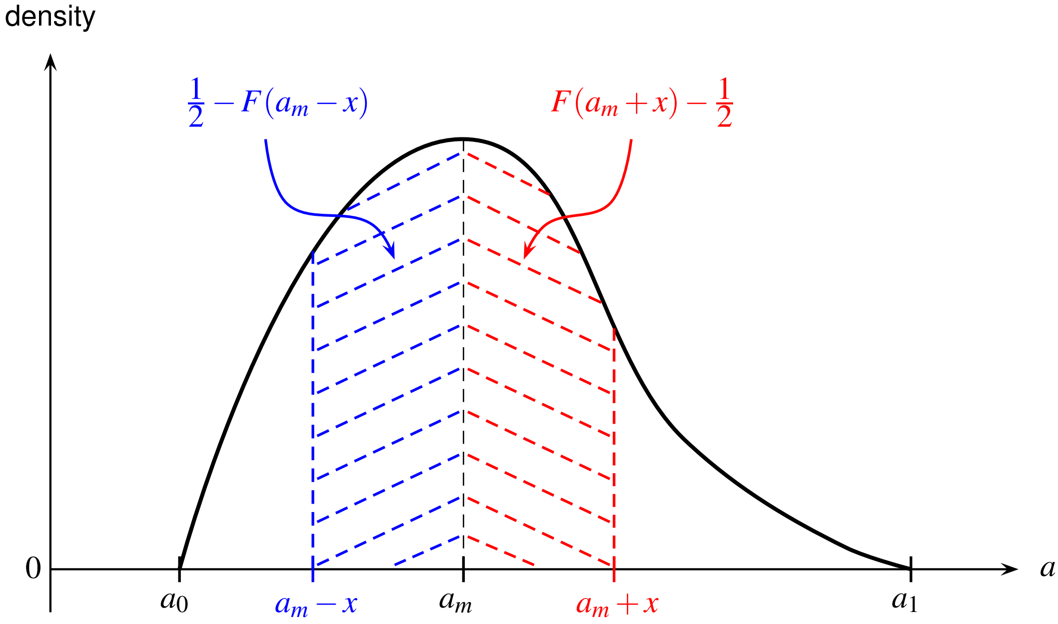
The bold black curve shows the marginal cost of effort at the median preferred standard $c_e(s_m, a)$ as a function of ability. The horizontal line represents the marginal benefit of an increase in the standard. The braces illustrate the loss and gain procured by such an increase to two individuals with abilities x units below and above the median. Condition 3 requires that the gain of the more able individual (right brace) is at least as large as the loss of the less able one (left brace).

symmetrically around the median. The condition requires that the mass of agents with abilities between the lower one of these values and the median is at least as large as the mass of agents with abilities between the median and the higher one of these values.

To summarise, Conditions 1 and 3 represent the idea that the impact of ability on marginal effort cost should be stronger on the high side of the ability distribution than on the low side. That is, academic performance is very sensitive to ability when one compares good and very good students, whereas below the median ability, differences in ability matter less. It is an empirical issue whether such a property holds in reality. A priori, it seems plausible to me because, on the one hand, weak students mostly can reach a satisfactory performance with sufficient training, whereas, on the other hand, really excellent achievements are out of reach except for the very brightest.

According to Conditions 2 and 4, the distribution of abilities is more 'spread-out' at the upper end of the support than at the lower end. This can arise, for example, by the presence in the economy of a few agents with very high ability, who raise the mean, whereas a large mass of agents is concentrated at moderately low ability levels. This corresponds to the empirical

Figure 4: Higher abilities are more spread out than lower abilities (Condition 4)



The curve illustrates an ability distribution which satisfies Condition 4. Starting from median ability, the probability mass covered by moving distance x to the left (shaded blue) is at least as large as the mass covered by moving the same distance to the right (shaded red).

fact that income distributions, which at least partially reflect distributions of productivity or ability, are typically right-skewed (for the U.S., see Guzman and Kollar, 2024, p. 16).

Proposition 2 *If Conditions 1 and 2 hold, then* $W'(s_m) \geq 0$. *If in addition, an inequality in one of these conditions is strict or* $a_{\min}(s_m) > a_o$, *then* $W'(s_m) > 0$.

Proof. See Appendix A.IV in the supplementary material. ∎

Proposition 3 *If Conditions 3 and 4 hold, then* $W'(s_m) \geq 0$. *If in addition, an inequality in one of these conditions is strict or* $a_{\min}(s_m) > a_o$, *then* $W'(s_m) > 0$.

Proof. See Appendix A.V in the supplementary material. ∎

Propositions 2 and 3 show that democratic choice leads to an inefficiently low examination standard if the cost function and the distribution function satisfy one of the pairs of Conditions 1 and 2, or 3 and 4. Intuitively, an increase in standard is beneficial if the gain conferred this way to agents whose abilities exceed the median by a certain amount outweighs the loss incurred by agents whose abilities fall short of the median by a similar amount. This is the case if the marginal cost of effort decreases fast once ability is raised above the median but rises only slowly when ability falls below the median, as required by Conditions 1 or 3. Moreover, the aggregate gain (loss) is large (small) if the mass of agents with very high (low) ability is relatively large (small), as postulated in Conditions 2 or 4.

Looking closer at the pairs of conditions required in each proposition, one notices that Condition 1 implies Condition 3, and that Condition 4 implies Condition 2. Therefore, there is a substitutive relationship between the properties of the cost function and the distribution function in the sense that it is possible to weaken one of them if one strengthens the other.

As mentioned in the introduction, Costrell (1994, p. 963-964) proves a result which appears to be contrary to Propositions 2 and 3. In his model, the standard chosen by majority vote is inefficiently high if the distribution of the standards preferred by voters is symmetric unimodal.

There are two major differences between both models. Technically, the present analysis uses conditions on the distribution of abilities and on the shape of the effort cost function instead of the distribution of preferred standards. Economically, both models assume different objective functions of voters: In Costrell (1994), voters care only about academic performance or productivity but do not take effort cost into account. In contrast, in the present analysis, it is assumed that parents will vote for reducing the standard if they feel that their children suffer too much from the effort required in school. Therefore, in Costrell (1994) the median chooses the maximal standard $s_{\max}(a_m)$ her child will meet, whereas here she chooses the optimal standard $s_m = s(a_m)$.

One can link both frameworks by using, as in Costrell (1994), aggregate productivity $V(s) = s\left[1 - F\left(a_{\min}(s)\right)\right]$ as the social objective and assuming that each voter prefers the maximal standard $s_{\max}(a)$. In Appendix A.VI(i) in the supplementary material it is shown that $V'(s_{\max}(a_m)) < 0$, i.e., the median preferred standard $s_{\max}(a_m)$ is too tough, if the distribution of maximal standards $s_{\max}(a)$ is symmetric and unimodal. Thus, Costrell's result can be replicated in the present model.

There is, however, no easy way to link the shape of the distribution of maximal standards $s_{\max}(a)$ to properties of the underlying distribution of abilities $F(a)$ and effort cost function

$c(e,a)$. In contrast, symmetry of the distribution of optimal standards $s(a)$ is induced, for example, if $F(a)$ is symmetric and all third derivatives of $c(e,a)$ are zero (see Appendix A.VI(ii) in the supplementary material). Under these assumptions, by the arguments in the proofs of Proposition 2 or 3, one finds $W'(s_m) \geq 0$. Hence, in the present model symmetry of the distribution of preferred standards does not imply an excessively tough standard. This highlights the importance of the main economic difference between both models, which is the objective assumed for voters. An education system where effort cost of students is politically relevant is likely to be less demanding than a system which only aims at raising educational outcomes.

In Propositions 2 and 3 the existence of agents who do not graduate under the median preferred standard figures only as a tie-breaking device in case both of the respective conditions are just satisfied as equalities. In the following Section 5, I show that the presence of a substantial number of drop-outs independently contributes to an insufficiently high median preferred standard. Hence, the pairs of conditions used in each proposition are sufficient for this result but not necessary.

5 The Role of Dropouts

By not graduating, students have the opportunity to avoid costly learning effort. Therefore, the possibility to drop out mitigates the negative welfare effect of a rising standard. As a result, in the presence of drop-outs, the median preferred standard may be too low even when Conditions 1 and 3 fail, that is, when marginal cost of effort is very high for low ability agents.

I will illustrate this effect by means of an example. In this example, ability is uniformly distributed on $A = [a_o, a_1] = [0,2]$ with $a_m = \overline{a} = 1$. Effort cost is given by a family of functions

$$c(e,a;\gamma) = \frac{e^2}{2}\left[1 + (a_m - a) + \gamma(a_m - a)^2\right], \tag{3}$$

where the parameter is restricted to $0 \leq \gamma \leq 1/2$ to ensure that $c(e,a;\gamma)$ satisfies Assumptions 1 to 3. Computing $s_m = 1$, $c_e(s_m,a;\gamma) = 2 - a + \gamma(1-a)^2$, $c_{ea}(s_m,a;\gamma) = -1 - 2\gamma(1-a)$, and $c_{eaa}(s_m,a;\gamma) = 2\gamma$, one sees that γ determines the curvature of the marginal cost of effort.

Specifically, for $\gamma > 0$ the example violates both Conditions 1 and 3.

The graduation threshold $a_{\min}(s_m;\gamma)$ solves the equation $c(s_m, a;\gamma) = s_m$, or equivalently, $1 + (a_m - a) + \gamma(a_m - a)^2 = 2/s_m$. With $a_m = s_m = 1$ it follows that for all admissible γ, the marginal cost of effort at the graduation threshold is $c_e(s_m, a_{\min}(s_m;\gamma);\gamma) = 2$. The change in welfare induced by a marginal increase in the standard can be computed from (2) as

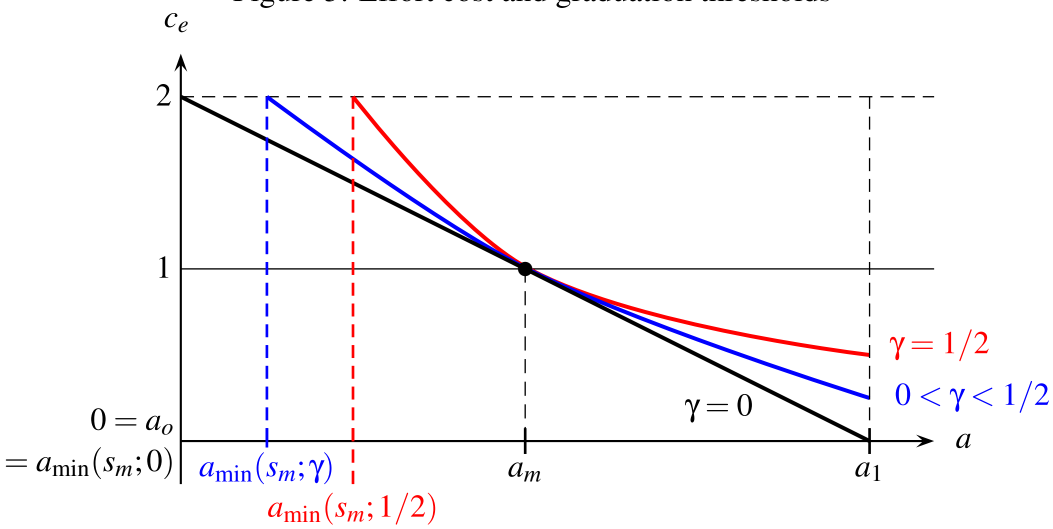
$$\frac{\partial W(s_m;\gamma)}{\partial s} = -\frac{1}{2}\int_{a_{\min}(s_m;\gamma)}^{2} \left[(1-a) + \gamma(1-a)^2\right] da\,.$$

From these equations, one derives:

Proposition 4 *If the cost of effort is given by (3) and ability is distributed uniformly on* $[0,2]$*, then* $\partial W(s_m;\gamma)/\partial s > 0$ *for all* $0 < \gamma < 1/2$.

Proof. See Appendix A.VII in the supplementary material. ∎

Figure 5: Effort cost and graduation thresholds



The downward sloping lines display marginal cost of effort at the median preferred standard $c_e(s_m, a;\gamma) = 2 - a + \gamma(1-a)^2$ as a function of ability for the lowest (black straight line), highest (higher curve, red), and an intermediate (lower curve, blue) curvature parameter γ. The horizontal line represents the marginal benefit of an increase in the standard. As marginal effort cost bend upwards, the difference between marginal benefit and marginal cost decreases for those agents who still graduate. At the same time, the graduation threshold rises. This mitigates the loss of low ability agents such that overall, increasing the standard above s_m raises welfare.

The logic of Proposition 4 is illustrated in Figure 5. Here, the downward sloping straight line is the marginal cost of effort for the lowest admissible value $\gamma = 0$, which leads to the graduation threshold $a_{\min}(s_m;0) = a_o = 0$. When γ rises above zero, the marginal cost of effort bends upwards and becomes strictly convex, as seen in the blue curve. The highest possible $\gamma = 1/2$ finally results in the highest curve, painted red. With uniform distribution of ability, the aggregate losses and gains of an increase in standard are directly measured by the areas between these curves and the marginal benefit of 1. It is apparent that the net gain would decrease in γ if the graduation threshold remained at $a_o = 0$. However, the threshold moves to the right as γ increases, so that the area representing the loss, which is bounded below by the vertical line at $a_{\min}(s_m;\gamma)$, shrinks.

6 Conclusion

This paper investigates direct democratic votes on a graduation standard, that is, a performance level required from students to pass an examination. Welfare properties of the median preferred standard are analysed. It is shown that the standard chosen by a majority of voters is less demanding than the standard which maximises a utilitarian welfare criterion if two conditions are satisfied. The first of these conditions requires that the marginal effort cost of learning decreases rapidly as one moves towards more able individuals. The second condition states that the distribution of abilities is right-skewed. When these two properties hold, there is much to gain from inducing more able individuals to exert more effort, and hence welfare increases if the standard is raised beyond the one preferred by the median voter. These results explain why democracies may find it hard to raise academic achievements when parents care for the effort cost their children experience at school.

It is worthwhile to discuss two effects not included in the model which may possibly counteract the tendency of democratic education policy towards overly lenient standards. The first such feature is the fact that turnout is generally lower among low income voters than in the general electorate. Inasmuch as income and ability are correlated, this otherwise deplorable fact tends to raise median ability among voters and hence works in favour of a higher standard. Second, a tax-transfer scheme may give low ability agents, who would be recipients of transfers, a stake in higher standards since these will raise wages and tax revenues. These examples illustrate that further research on the political economics of graduation standards is worthwhile.

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Supplementary Material

Majority Vote on Educational Standards

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A.I Proof of Proposition 1

Consider s_m in a vote against some standard $s < s_m$. For $a > a_m$, one has $s_m < s(a)$, and hence $v(s,a)$ is strictly increasing in the standard between s and s_m (see Figure 1). This implies $v(s_m,a) > v(s,a)$ for all $a \geq a_m$. By continuity of $v(s,a)$ and since the density of the ability distribution is strictly positive, there is, in addition, a strictly positive mass of agents with $a < a_m$ who also strictly prefer s_m to s. Thus, $\int_{\{a \in A | v(s_m,a) > v(s,a)\}} dF(a) > 1/2$ follows.

Consider now s_m in a vote against some standard $s' > s_m$. By an analogous argument, one now has $v(s_m,a) \geq v(s_m,a)$ for all $a \leq a_m$, where equality arises (only) if $a < a_{\min}(s_m)$. Again by continuity, a positive mass of agents with ability $a > a_m$ also prefer s_m to s', and $\int_{\{a \in A | v(s_m,a) \geq v(s',a)\}} dF(a) > 1/2$ is proved. ∎

A.II Non-uniqueness of Condorcet winner

In this Appendix I provide a sufficient condition for a standard $\tilde{s} \neq s_m$ to be a Condorcet winner. Such a standard cannot be less demanding than the median preferred standard or so high that the median will not satisfy it. To see why, observe first that all agents with ability $a > a_m$ would graduate under both $\tilde{s} < s_m$ and s_m, and hence would necessarily support s_m in a vote between these two standards. Second, a standard $\tilde{s} \geq s_{\max}(a_m)$ would lead to zero utility for the median and all agents with lower ability $a < a_m$. However, with a small enough positive standard, positive utility is feasible for all agents, and hence $\tilde{s}$ would lose a vote against such an alternative.

Consider then $s_m < \tilde{s} < s_{\max}(a_m)$, and denote by $\tilde{a} := s^{-1}(\tilde{s})$ the ability of agents for whom $\tilde{s}$ is the optimal standard. Moreover, denote by s_o the standard which yields the same indirect utility for the median as $\tilde{s}$, defined by $v(s_o, a_m) = v(\tilde{s}, a_m)$.

The key element in the argument is that in all votes opposing $\tilde{s}$ to some other standard s, the tie of indifferent agents is broken in favour of $\tilde{s}$. There are three cases. (1) If $\tilde{s} < s$, then all agents with ability $a < \tilde{a}$ weakly or strongly prefer $\tilde{s}$. Since $a_m < \tilde{a}$, this is the majority. (2) If $0 \leq s < s_o$, all agents with $a \geq a_m$ strictly prefer $\tilde{s}$, and hence $\tilde{s}$ obtains a majority.

The critical case is (3): $s_o \leq s < \tilde{s}$. (Note that this interval contains s_m.) Since $a_{\min}(\cdot)$ is an increasing function, we have $a_{\min}(s_o) \leq a_{\min}(s)$. Hence, all agents with ability $a \leq a_{\min}(s_o)$ drop out under both standards and so are indifferent. Moreover, since $s(\cdot)$ is increasing, we have $s < \tilde{s} \leq s(a)$ for agents with $a \geq \tilde{a}$, hence all such agents strictly prefer $\tilde{s}$. Both groups of agents together ensure a majority for $\tilde{s}$ if $F(a_{\min}(s_o)) + 1 - F(\tilde{a}) > 1/2$, or equivalently

$$F\left(a_{\min}(s_o)\right) > F\left(\tilde{a}\right) - \frac{1}{2}. \tag{A.1}$$

The following numerical example shows that the sufficient condition (A.1) is consistent with the assumptions of the model. Let ability follow a triangular distribution on the support $[a_o, a_1) = [0, 2\sqrt{2})$ with c.d.f. $F(a) = a^2/8$. Median ability is $a_m = 2$. The learning cost function is assumed to be $c(e,a) = \left(e^2/2\right) \cdot (3-a)$. This yields optimal standards $s(a) = 1/(3-a)$ and graduation thresholds $a_{\min}(s) = 3 - (2/s)$. It follows $s_m = 1$, $a_{\min}(s_m) = 1$, and $\tilde{a} = 3 - (1/\tilde{s})$. Solving $s_o - \left(s_o^2/2\right) \cdot (3 - a_m) = \tilde{s} - \left(\tilde{s}^2/2\right) \cdot (3 - a_m)$ one finds $s_o = 2 - \tilde{s}$, and hence $a_{\min}(s_o) = 3 - [2/(2-\tilde{s})]$. After inserting into $F(\cdot)$, condition (A.1) becomes

$$\frac{1}{8} \cdot \left(3 - \frac{2}{2-\tilde{s}}\right)^2 > \frac{1}{8} \cdot \left(3 - \frac{1}{\tilde{s}}\right)^2 - \frac{1}{2}.$$

It can be verified numerically that this inequality is satisfied for $s_m < \tilde{s} < 1.1326$.

A.III Refinement of Condorcet equilibrium

In this appendix, I propose a refinement of the concept of a Condorcet winner in the spirit of trembling hand perfection. For this purpose, I define an ε-education model where for every standard $s \in \mathbb{R}_{\geq 0}$ each of the two options '$e = s$' and '$e = 0$' will be chosen with probability of at least $\varepsilon > 0$, where ε is a small number. In an ε-education model, the payoff from standard s for an agent with ability a will be $v(s,a;\varepsilon) = (1-\varepsilon)[s - c(s,a)]$ if $s - c(s,a) \geq 0$ and $v(s,a;\varepsilon) = \varepsilon[s - c(s,a)]$ if $s - c(s,a) < 0$. Adapting Definition 2 and Proposition 1 to the ε-education model, one obtains

Definition A.1 *Standard $s \in \mathbb{R}_{\geq 0}$ is a Condorcet winner in the ε-education model if for all standards $s' \in \mathbb{R}_{\geq 0}, s' \neq s$:*

$$\int_{\{a \in A | v(s,a;\varepsilon) \geq v(s',a;\varepsilon)\}} dF(a) > 1/2.$$

Lemma A.1 *Consider an ε-education model where agents with probability ε make an error in their graduation decisions, with $0 < \varepsilon < 1$. In that model, the standard s_m preferred by agents with median ability is the unique Condorcet winner.*

Proof: From $1 - \varepsilon > 0$, $v(s,a;\varepsilon) = (1-\varepsilon)[s - c(s,a)]$ is strictly increasing (strictly decreasing) in s for $0 \leq s < s(a)$ (for $s(a) < s < s_{\max}(a)$). From $\varepsilon > 0$, $v(s,a;\varepsilon) = \varepsilon[s - c(s,a)]$ is still strictly decreasing in s for $s > s_{\max}(a)$. Therefore, in a vote among s_m and a lower (higher) standard $s < s_m$ ($s' > s_m$), all agents with $a \geq a_m$ ($a \leq a_m$) strictly prefer s_m over s (over s'). By continuity of $v(s,a;\varepsilon)$ and from positive density of ability, this holds also for a positive mass of agents with $a < a_m$ ($a > a_m$). Hence, s_m is strictly preferred to s (s') by more than half of the electorate. Therefore, s_m beats every other standard, and no other standard can attract a majority of voters against s_m. ∎

According to the idea of trembling hand perfection, a Condorcet winner in the original model is only reasonable if it is robust against the possibility of small errors. This is captured by the following definition:

Definition A.2 *A standard $s \in \mathbb{R}_{\geq 0}$ is a strong Condorcet winner if*

(i) s is a Condorcet winner, and

(ii) there is a sequence $\{s_n\}_{n=1,2,...}$ such that $s_n \to s$ and for all n, s_n is a Condorcet winner in an ε_n-education model, where $\varepsilon_n \in (0,1)$ and $\varepsilon_n \to 0$.

One immediately concludes from Proposition 1 and Lemma A.1:

Proposition A.1 *The standard s_m preferred by agents with median ability is the unique strong Condorcet winner.*

A.IV Proof of Proposition 2

(i) For brevity, define $\beta := -c_{ea}(s_m, a_m) > 0$. With this, from Condition 1, one has

$$c_e(s_m, a) \leq c_e(s_m, a_m) + \beta(a_m - a) \tag{A.2}$$

for all $a \in A$ (see Figure 2), and, since $c_e(s_m, a_m) = 1$, it follows $c_e(s_m, a) \leq 1 + \beta(a_m - a)$ for all $a \in A$. Inserting into (2), one obtains

$$\begin{aligned} W'(s_m) &\geq \beta \int_{a_{\min}(s_m)}^{a_1} (a - a_m)\, dF(a) \\ &= \beta\left[1 - F(a_{\min}(s_m))\right] \cdot \left[E\left(a | a > a_{\min}(s_m)\right) - a_m\right] , \end{aligned} \tag{A.3}$$

where $E\left(a|a > a_{\min}(s_m)\right) = \int_{a_{\min}(s_m)}^{a_1} a\, dF(a) / \left[1 - F(a_{\min}(s_m))\right]$ is the expected ability of graduates, which is well defined since under the median preferred standard, a positive mass of agents will graduate. Clearly, $E\left(a|a > a_{\min}(s_m)\right) \geq \overline{a}$, and hence (A.3) implies

$$W'(s_m) \geq \beta\left[1 - F(a_{\min}(s_m))\right] \cdot \left[\overline{a} - a_m\right] . \tag{A.4}$$

From this and Condition 2, it follows $W'(s_m) \geq 0$.

(ii) If the inequality in Condition 1 is strict, then (A.2), and by consequence (A.3) hold with strict inequality. If the inequality in Condition 2 is strict, then the right-hand-side of (A.4) is strictly positive. If $a_{\min}(s_m) > a_o$, then $E\big(a|a > a_{\min}(s_m)\big) > \overline{a}$ so that in (A.3), the right-hand-side is strictly positive. In all three cases, it follows $W'(s_m) > 0$. ■

A.V Proof of Proposition 3

(i) Splitting the integral in (2) at the median and writing $a_m - x = a$ for $a < a_m$ and $a_m + x = a$ for $a > a_m$, one obtains

$$W'(s_m) = -\int_{a_{\min}(s_m)}^{a_m} \big[c_e(s_m, a_m - x) - 1\big]\, dF(a_m - x) + \int_{a_m}^{a_1} \big[1 - c_e(s_m, a_m + x)\big]\, dF(a_m + x)\,. \tag{A.5}$$

From Condition 4, it must hold $a_m - a_o \leq a_1 - a_m$ so that Condition 3 implies

$$c_e(s_m, a_m - x) - 1 \leq 1 - c_e(s_m, a_m + x) \tag{A.6}$$

for all $x \in (0, a_m - a_o]$. Using (A.6) in (A.5), one concludes

$$W'(s_m) \geq -\int_{a_{\min}(s_m)}^{a_m} \big[1 - c_e(s_m, a_m + x)\big]\, dF(a_m - x) + \int_{a_m}^{a_1} \big[1 - c_e(s_m, a_m + x)\big]\, dF(a_m + x)\,. \tag{A.7}$$

Define the two distribution functions $G(x) := 1 - 2F(a_m - x)$ for $x \in [0, a_m - a_o]$ and $H(x) := 2F(a_m + x) - 1$ for $x \in [0, a_1 - a_m]$. ($G(x)$ resp. $H(x)$ is the probability that ability is at most a distance x away from the median, conditional on being below resp. above the median.) One has $dG(x) = -2dF(a_m - x)$ and $dH(x) = 2dF(a_m + x)$. Using G and H in (A.7), adjusting the integration bounds appropriately and reversing the order of integration in the first integral, one arrives at

$$\begin{aligned} W'(s_m) \geq -\frac{1}{2}\int_0^{a_m - a_{\min}(s_m)} \left[1 - c_e(s_m, a_m + x)\right] dG(x) \\ + \frac{1}{2}\int_0^{a_1 - a_m} \left[1 - c_e(s_m, a_m + x)\right] dH(x)\,. \end{aligned}$$

Since $a_{\min(s_m)} \geq a_o$ and $1 - c_e(s_m, a_m + x) > 0$ for $x > 0$, it follows furthermore

$$\begin{aligned} W'(s_m) \geq -\frac{1}{2}\int_0^{a_m - a_o} \left[1 - c_e(s_m, a_m + x)\right] dG(x) \\ + \frac{1}{2}\int_0^{a_1 - a_m} \left[1 - c_e(s_m, a_m + x)\right] dH(x)\,. \end{aligned} \tag{A.8}$$

Now observe that the two integrals in (A.8) have the form of expected utilities, with $1 - c_e(s_m, a_m + x)$ as the utility function which strictly increases in the random variable x because of Assumption 3. Moreover, Condition 4 implies that the distribution $H(x)$ first order stochastically dominates the distribution $G(x)$. Since a decision-maker with monotonic preferences will prefer the dominating to the dominated lottery (see Yildiz, 2015, Theorem 4.1), the second integral must be at least as large as the first one. This implies $W'(s_m) \geq 0$.

(ii) If the inequality in Condition 3 is strict, then (A.6) and hence (A.7) hold as strict inequalities. If the inequality in Condition 4 is strict, the dominance of the second over the first integral in (A.8) is strict, implying that the right-hand-side of this inequality is strictly positive. Finally, if $a_{\min}(s_m) > a_o$, extending the integration in (A.8) to the values $x \in (a_m - a_{\min}(s_m), a_m - a_o]$ adds a positive mass of strictly negative values, so that (A.8) holds as a strict inequality. In all three cases, it follows $W'(s_m) > 0$. ■

A.VI Symmetric distribution of preferred standards

(i) Differentiating $V(s)$ yields $V'(s) = 1 - F\left(a_{\min}(s)\right) - s \cdot f\left(a_{\min}(s)\right) \cdot da_{\min}(s)/ds$, where $f(a) := F'(a)$ is the density of the ability distribution. Evaluating this at $s_{\max}(a_m)$ and observing $a_{\min}(s_{\max}(a_m)) = a_m$, one finds

$$V'(s_{\max}(a_m)) = \frac{1}{2} - s_{\max}(a_m) \cdot f(a_m) \cdot \frac{da_{\min}(s_{\max}(a_m))}{ds} \; . \tag{A.9}$$

The distribution of maximal standards is given by

$$\text{Prob}\{s_{max}(a) \leq s\} = \text{Prob}\{a_{min}(s) \geq a\} = F(a_{\min}(s)) \; ,$$

with density $f(a_{\min}(s)) \cdot da_{\min}(s)/ds$. If this distribution is symmetric unimodal, one has $s_{\max}(a_m) = [s_{\max}(a_o) + s_{\max}(a_1)]/2$ and $f(a_m) \cdot da_{\min}(s_{\max}(a_m))/ds \geq 1/[s_{\max}(a_1) - s_{\max}(a_o)]$. Using this and $s_{\max}(a_o) > 0$ in (A.9), once concludes $V'(s_{\max}(a_m)) < 0$.

(ii) If all third derivatives of the cost function are zero, the marginal cost of effort can be written as $c_e(s,a) = c_e(s_m, a_m) + \beta(a_m - a) + \varphi(s - s_m)$, with positive constants $\beta > 0$ and $\varphi > 0$. Using $c_e(s_m, a_m) = 1$, the optimal standard of an agent with ability a is then $s(a) = s_m + (\beta/\varphi)(a - a_m)$. The distribution of optimal standards is given by

$$\text{Prob}\{s(a) \leq s\} = \text{Prob}\left\{s_m + \frac{\beta}{\varphi}(a - a_m) \leq s\right\} = F\left(a_m + \frac{\varphi}{\beta}(s - s_m)\right) \; .$$

With symmetric $F(a)$, it follows that for all $y \in [0, s_m - s(a_o)] = [0, s(a_1) - s_m]$,

$$\begin{aligned} \text{Prob}\{s_m - y \leq s(a) \leq s_m\} &= \frac{1}{2} - F\left(a_m - \frac{\varphi}{\beta}y\right) \\ &= F\left(a_m + \frac{\varphi}{\beta}y\right) - \frac{1}{2} \\ &= \text{Prob}\{s_m \leq s(a) \leq s_m + y\} \; . \end{aligned}$$

Thus, the distribution of optimal standards is symmetric.

A.VII Proof of Proposition 4

Computations done with Mathematica (see next pages) reveal that the only two values of $\gamma \in [0,1/2]$ with $\partial W(s_m;\gamma)/\partial s = 0$ are $\gamma = 0$ and $\gamma = 1/2$, and that $\partial W(s_m;\gamma)/\partial s$ is increasing in γ at $\gamma = 0$. From this, it follows $\partial W(s_m;\gamma)/\partial s > 0$ for $0 < \gamma < 1/2$. ■

In[∘]:=
```
(* MAJORITY VOTE ON EDUCATIONAL STANDARDS -- Proof of Proposition 4 *)

(* effort cost function, with x for effort,
and median ability am=1 inserted *)
K[x_, a_, g_] := (x^2 / 2) * (2 - a + g (1 - a) ^2)
(* marginal cost of effort for any ability a*)
D[K[x, a, g], x]
```
leite ab

Out[∘]=

$$\left(2-a+(1-a)^2 g\right) x$$

In[∘]:=
```
(* define marginal cost of effort c_e = MK *)
MK[x_, a_, g_] := (2 - a + (1 - a)^2 g) x
(* find optimal standard for ability a *)
Solve[MK[x, a, g] == 1, x]
```
löse

Out[∘]=

$$\left\{\left\{x \to \frac{1}{2-a+g-2 a g+a^2 g}\right\}\right\}$$

In[∘]:=
```
(* define optimal standard s(a,g) = S[a_,g_] *)
S[a_, g_] := 1 / (2 - a + g - 2 a g + a^2 g)
(* optimal standard for median sm = S[1,g]*)
sm := S[1, g]
sm
```

Out[∘]=

1

In[∘]:=
```
(* find graduation threshold amin
  (sm,g) for median preferred standard sm=1 *)
Reduce[K[1, a, g] == 1, a]
```
reduziere

Out[∘]=

$$(g == 0 \,\&\&\, a == 0) \;||\; \left(g \neq 0 \,\&\&\, \left(a == \frac{1+2g-\sqrt{1+4g}}{2g} \;||\; a == \frac{1+2g+\sqrt{1+4g}}{2g}\right)\right)$$

In[∘]:=
```
(* check continuity at g=0 *)
Limit[(1 + 2 g - Sqrt[1 + 4 g]) / (2 g), g -> 0]
```
Grenzwert

Out[∘]=

0

In[∘]:=
```
(* define graduation threshold amin(sm,g) = Amin[g],
as function of curvature parameter g *)
Amin[g_] := Piecewise[{{0, g == 0}, {(1 + 2 g - Sqrt[1 + 4 g])/(2 g), g ≠ 0}}]
```
stückweise

$$\texttt{Amin}[g\_] := \texttt{Piecewise}\left[\left\{\{0,\ g == 0\},\ \left\{\frac{1+2g-\sqrt{1+4g}}{2g},\ g \neq 0\right\}\right\}\right]$$

```
(* verify marginal cost of effort at graduation threshold *)
Simplify[MK[sm, Amin[g], g]]
```
vereinfache

Out[∘]=

2

In[∘]:=
```
(* derivative of welfare with respect to standard d W(sm,g)/d s;
at median preferred standard sm,
with density 1/2 and graduation threshold Amin(g) *)
Simplify[Integrate[(1 - MK[sm, a, g]) * (1 / 2), {a, Amin[g], 2}]]
```
vereinfache integriere

Out[∘]=

$$\begin{cases} -\frac{g}{3} & g == 0 \\ -\frac{(-1+2g)\left(-1-2g+2g^2+\sqrt{1+4g}\right)}{24 g^2} & \text{True} \end{cases}$$

In[∘]:=
```
(* check continuity at g=0 *)
```

$$\texttt{Limit}\left[-\frac{(-1+2g)\left(-1-2g+2g^2+\sqrt{1+4g}\right)}{24 g^2},\ g \to 0\right]$$

Grenzwert

Out[∘]=

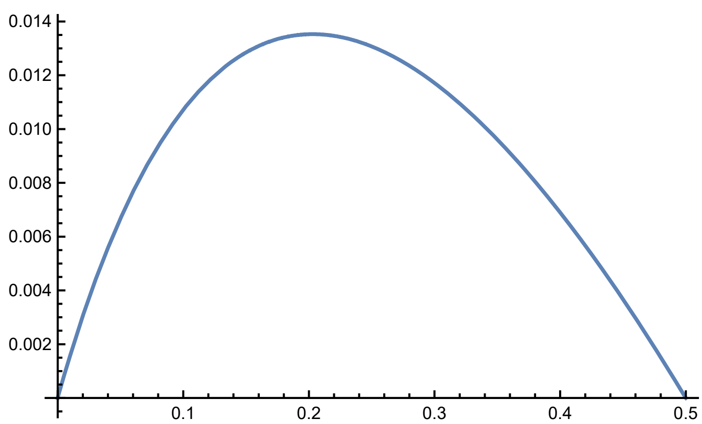
0

In[∘]:=
```
(* define d W(sm,g)/d s  = Wprime *)
Wprime[g_] := Integrate[(1 - MK[sm, a, g]) * (1 / 2), {a, Amin[g], 2}]
```
integriere

```
(* find zeros of d W(sm,g)/d s *)
Plot[Wprime[g], {g, 0, 1 / 2}]
```
stelle Funktion graphisch dar

```
Solve[Wprime[g] == 0, g]
```
löse

Out[∘]=



Out[∘]=

$$\left\{\{g \to 0\},\ \left\{g \to \frac{1}{2}\right\}\right\}$$

In[∘]:= (* compute slope of d W(sm,g)/d s w.r.t. g *)

$$\text{Simplify}\left[D\left[-\frac{(-1+2g)\left(-1-2g+2g^2+\sqrt{1+4g}\right)}{24g^2}, g\right]\right]$$

vereinfache leite ab

Out[∘]=

$$\frac{-1-2g+2g^2+\sqrt{1+4g}-2g^3\sqrt{1+4g}}{12g^3\sqrt{1+4g}}$$

In[∘]:= (* verify that d W(sm,g)/d s is increasing at g=0 *)

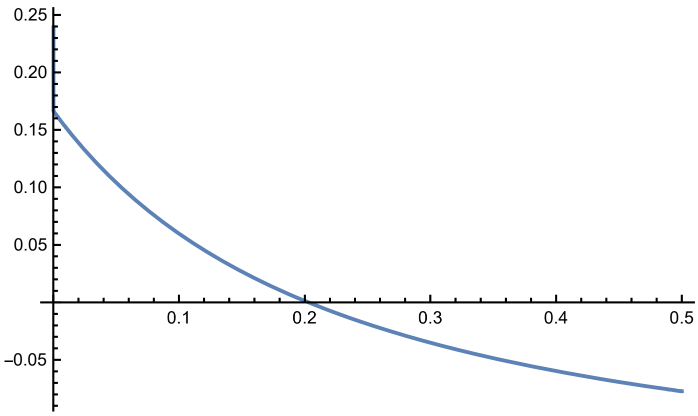
$$\text{Plot}\left[\frac{-1-2g+2g^2+\sqrt{1+4g}-2g^3\sqrt{1+4g}}{12g^3\sqrt{1+4g}}, \{g, 0, 1/2\}\right]$$

stelle Funktion graphisch dar

$$\text{Limit}\left[\frac{-1-2g+2g^2+\sqrt{1+4g}-2g^3\sqrt{1+4g}}{12g^3\sqrt{1+4g}}, g \to 0\right]$$

Grenzwert

Out[∘]=



Out[∘]=

$$\frac{1}{6}$$

In[∘]:= ClearAll[a, g, x, sm]

lösche alle

References

YILDIZ, M. (2015): "14.123 Microeconomic Theory III," Massachusetts Institute of Technology: MIT OpenCourseWare, https://ocw.mit.edu, accessed August 8, 2021.