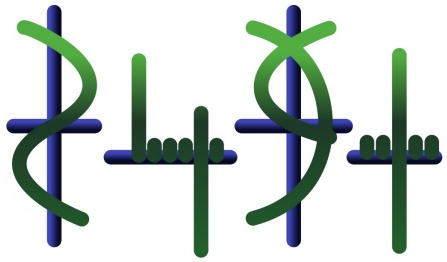
# RTG2491 Spring School

# Decoupling and More

March 24–28, 2025




Information, schedule, & abstracts

Scientific Organizers:

Damaris Schindler (Göttingen, Germany)
Christian Jäh (Göttingen, Germany)
Tanja Küfner (Göttingen, Germany)
Anouk Greven (Göttingen, Germany)

Administrative Organizers:

Linda Haber (Göttingen, Germany)
Annalena Wendehorst (Göttingen, Germany)

Contents

## 1. General Schedule

- The Q & A sessions are reserved for in-depth questions and discussions on the presented material. You can also use the **Schlauch** (behind the Sitzungszimmer) to have discussions and contemplate the material. The lectures will all be in the **Sitzungszimmer**.
- The conference dinner will be at Le Feu (Menu, Google maps). The food (buffet) and water for the table are included; drinks are on you. It might be good to bring some cash.[1]

Schedule Decoupling and more, March 2025

| *time* | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| 09.00 - 09.30 | *Registration* | | | | |
| 09.30 - 10.30 | Zane Kun Li | Niclas Technau | Katy Woo | Zane Kun Li | Changkeun Oh |
| 10.30 - 11.00 | *Coffee break* | | | | |
| 11.00 - 12.00 | Gong talks | Rajula Srivastava | Zane Kun Li | Gong talks | Niclas Technau |
| 12.00 - 14.00 | *LUNCH* | | | | |
| 14.00 - 15.00 | Niclas Technau | Zane Kun Li | Free afternoon | Changkeun Oh | |
| 15.00 - 15.30 | *Coffee break* | | | *Coffee break* | |
| 15.30 - 16.30 | Question session | Changkeun Oh | | Question session | |
| | | | | Dinner at Le Feu | |

[1] Many businesses in Germany are still in the stone age as far as the option of paying not in cash are concerned. Check!

## 2. Gong Talks Schedule

| Session | Speakers |
|---|---|
| Monday 11 -12 | Tammo Dede, Matteo Brodignon, Eduard Stefanescu |
| Thursday 11 - 12 | Annemily Hoganson, Lars Becker, Hrit Roy, Cameron Wilson |

## 3. Some introductory material

There are several good starting points for information on decoupling. For instance the lectures of Larry Guth

- 1/3 Introduction to decoupling [MSRI 2017]
- 2/3 Ingredients of the proof of decoupling [MSRI 2017]
- 3/3 The proof of decoupling for the parabola [MSRI 2017]

as well as the book *Fourier Restriction, Decoupling, and Applications* [1]. Here are also some nice Lecture Notes by Pavel Zorin-Kranich (U-Bonn).
Concerning the original proof of Bourgain and Demeter [2], the best starting point, maybe after the above mentioned lectures by Larry Guth, is the paper *A study guide for the $\ell^2$ decoupling theorem* [3]. More details can then be found in the original literature as well as Demeter's book [1]; here, also some applications to PDE and Number Theory can be found.

More directly connected links to work of the speakers in this school:

- Connected to the talks of Zane Kun Li (Section 4.1) you might find the notes *An introduction to decoupling and harmonic analysis over $\mathbb{Q}_p$* on arXiv (2209.01644) helpful.

**References.**


[1] Ciprian Demeter. *Fourier restriction, decoupling, and applications*. Vol. 184. Cambridge Studies in Advanced Mathematics. Cambridge University Press, Cambridge, 2020, pp. xvi+331. ISBN: 978-1-108-49970-5. DOI: 10.1017/9781108584401.

[2] Jean Bourgain and Ciprian Demeter. "The proof of the $\ell^2$ decoupling conjecture". In: *Ann. of Math. (2)* 182.1 (2015), pp. 351–389. DOI: 10.4007/annals.2015.182.1.9.

[3] Jean Bourgain and Ciprian Demeter. "A study guide for the $\ell^2$ decoupling theorem". In: *Chinese Ann. Math. Ser. B* 38.1 (2017), pp. 173–200. DOI: 10.1007/s11401-016-1066-1.

## 4. Abstracts – Mini Courses

### 4.1. **An introduction to decoupling theory.**

Zane Kun Li
North Carolina State University
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In the last 10 years, Fourier decoupling theory has had numerous striking applications to PDE, number theory, and geometric measure theory. In this series of lectures, we will provide an introduction to decoupling theory by discussing tools, heuristics, and techniques for Fourier decoupling, in particular concentrating on the case of decoupling for the parabola.

### 4.2. **A high-low method in decoupling theory.**

Changkeun Oh
Seoul National University
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In 2014, a decoupling inequality was proved by Bourgain and Demeter. Since their work, many different proofs of the decoupling inequality for the parabola have been found. Among them, the proof using the high-low method is particularly notable because it provides the current best bound for the decoupling constant for the parabola. In this series of lectures, I will give a simplified proof of a decoupling inequality for the parabola using a high-low method.

### 4.3. **Counting Rational Points Near Manifolds.**

Niclas Technau
Bonn Univeristy
ntechnau@uni-bonn.de

Counting rational points on, or close to manifolds is a basic problem with far reaching applications. The objective of this mini-course is twofold. First, we explain the role of Fourier transforms of surface measures for this counting problem. Special attention is paid to an inductive, multi-scale argument due to J.-J. Huang. Second, we detail applications in algebraic geometry (Serre's dimension growth conjecture), and Diophantine approximation (Khintchine's theorem on manifolds).

## 5. Abstracts – Contributed Talks

### 5.1. **Counting Rational Points near Manifolds: Beyond Hypersurfaces.**

Rajula Srivastava
Bonn Univeristy & Max Planck Institute for Mathematics
rajulas@math.uni-bonn.de.

How many rational points with denominator of a given size lie within a certain distance from a compact, "non-degenerate" manifold, of arbitrary codimension? This talk is about some recent progress towards answering this question, based on joint work with Damaris Schindler and Niclas Technau. We shall discuss a new way of examining the problem: under a combined lens of harmonic analysis (oscillatory integrals) and homogeneous dynamics (quantitative non-divergence estimates).

### 5.2. **Manin's conjecture for Chatelet surfaces.**

Katharine (Katy) Woo
Princeton University
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## 6. Abstracts – Gong Talks

These talks will be given in the gong sessions of the meeting.

### 6.1. **Small solutions to linear forms in primes.**

Tammo Dede
Georg-August-University Göttingen

We show that almost all linear forms in at least four variables admit a prime solution that is as small as heuristically expected.

### 6.2. **Weyl sums with multiplicative coefficients and joint equidistribution.**

Matteo Bordignon
KTH Royal Institute of Technology

We generalize a result of Montgomery and Vaughan regarding exponential sums with multiplicative coefficients to the setting of Weyl sums. As applications, we establish a joint equidistribution result for roots of polynomial congruences and polynomial values and obtain some new results for mixed character sums.

### 6.3. **The maximal volume of empty convex bodies amidst multivariate dilates of a lacunary integer sequence.**

Eduard Stefanescu
Graz University of Technology

Let $(a_n)_{n\in\mathbb{N}}$ be a Hadamard lacunary sequence. We establish upper bounds for the maximal volume of convex sets inscribing the set of dilates $\{a_n\boldsymbol{\alpha}\}_{n\leq N}$ modulo $\mathbf{1}$, in terms of $N$. For any lacunary sequence $(a_n)_{n\in\mathbb{N}}$, we prove that for Lebesgue-almost all $\boldsymbol{\alpha}\in[0,1]^d$, the maximal volume of empty convex bodies is at most of order $(\log N)^{2+\varepsilon}/N$. This metric result is further generalized to other measures satisfying a certain Fourier decay assumption. Both upper bounds are optimal up to a factor of logarithmic order, and the latter result improves and generalizes a recent result of Chow and Technau.

### 6.4. **Random Multiplicative Functions over Function Fields.**

Annemily Hoganson
Georg-August-University Göttingen

Granville-Soundararajan, Harper-Nikeghbali-Radziwill, and Heap-Lindqvist independently established an asymptotic for the even natural moments of partial sums of random multiplicative functions defined over integers. Building on these works, we study the even natural moments of partial sums of Steinhaus random multiplicative functions defined over function fields. Using a combination of analytic and combinatorial arguments, we obtain an asymptotic expression for

all the even natural moments in the large field limit and large degree limit, as well as an exact expression for the fourth moment.

6.5. **A discrete Brunn-Minkowski inequality for subsets of the cube.**

Lars Becker
Universität Bonn

This talk is about a variant of the classical Brunn-Minkowski inequality for sets in the lattice $Z^d$: If A, B are subsets of a cube, then $|A + B| > (|A||B|)^c$ for some exponent $c$ strictly larger than $\frac{1}{2}$. We determine the optimal value of $c$ in the special case where the cube has sidelength 2, and discuss a nice trick to prove polynomial inequalities which is used in the proof. This is based on joint work with Paata Ivanisvili, Dmitry Krachun and José Madrid.

6.6. **A Uniform Decoupling Estimate for Convex Curves in the Plane.**

Hrit Roy
University of Edinburgh

We discuss a universal $l^2L^6$ decoupling inequality for convex curves in the plane. These estimates are obtained using the high/low method. We also discuss examples of convex curves with special arithmetic structures that satisfy decoupling estimates stronger than the universal bound.

6.7. **Moments for the polynomial Chowla conjecture.**

Cameron Wilson
University of Glasgow

The polynomial Chowla conjecture makes predictions about the average of the Liouville function over values of fixed (admissible) polynomials. I will briefly review progress made towards this conjecture, and introduce some results describing how such averages behave for random polynomials.

## 7. Where to go for lunch?

For the Zentralmensa, you need a guest card to pay as there is no cash payment at the moment. You can get the guest card at the information desk at the mensa (on the first day, someone of us will help you) and you can then charge it on a machine with an EC card.

- Zentralmensa (Google maps). This is the main mesa on the central campus. This is about 20 minutes walk from the institute.[2] The menu can be found here (choose Zentralmensa in the drop-down). The menu is also displayed in English at the mensa.
  In the Café Zentral a selection of hot and cold sandwiches and pizzas as well as sweets is offered.
- Bakery Küster (here or here).
  They have a great selection of hot and cold sandwiches, salads, and other snacks. Not to forget sweets.
  Here is a link to the Cafe menu. (German)
- Bakery Holzofenbäckerei (Google maps).
  Here you can get sandwiches, soup, and some other snacks.
- Vietnamese Restaurant Nam Anh (Vietnamese, Google maps)
  Groner Str. 12, 37073 Göttingen
- Asian Fusion Gamie (Vietnamise/Japanese, Google maps)
  Weender Str. 29, 37073 Göttingen
- Bullerjahn (German/International, Google maps) Markt 9, 37073 Göttingen
- Empanadas Sabrosita (Spanish, Google maps)
  Karspüle 9, 37073 Göttingen

[2] Can be connected with a stroll to Gauß' grave on the way back.

## 8. Where to go for dinner?

Here is a list of reasonable restaurants:

- Kartoffelhaus (German, Google maps)
  Goetheallee 8, 37073 Göttingen, Menu (in German)
- Vietnamese Restaurant Nam Anh (Vietnamese, Google maps)
  Groner Str. 12, 37073 Göttingen
- Asian Fusion Gamie (Vietnamise/Japanese, Google maps)
  Weender Str. 29, 37073 Göttingen
- Bullerjahn (German/International, Google maps) Markt 9, 37073 Göttingen
- Abessina (Ethiopian restaurant, Google maps)
  Ritterplan 2, 37073 Göttingen

The institute can provide a list with further options and of course we will try to be as helpful in finding something suitable if you ask us.

8.1. **Vegan options.** Some of the restaurants named above have vegetarian and also some vegan options. There are however some more specialized options:

- Sen Viet (Vietnamese, Google maps)
- Goa India (Indian, Google maps)
- Chay Vegan Kitchen (Vietmaes ed al, Google maps)

## 9. The University and the city of Göttingen

Some sights/museums:

- The University has a lot of collections which you can brows here. In Mathematics, we have in particular a collection of models, calculating machines, integrators, etc. These can in part be viewed in the foyer in front of the room where the school takes place. Further information here.
- Forum Wissen – the new museum of knowledge in Göttingen – aims to make research with objects tangible., (Admission free.)
- Gänselisel on the main market place in front of the old city hall. PhD student traditionally bring her, following a successful defence, flowers and give her a kiss.
- Gauß' grave
- Gauß Observatory
- City Cemetery with graves of David Hilbert, Max Born, Otto Hahn, Max Planck and many more.

## 10. RTG 2491 - Fourier Analysis and Spectral Theory

The Research Training Group (RTG) Fourier Analysis and Spectral Theory is a joint research and graduate education programme funded by the German Science Foundation (DFG). It is based at the Mathematical Institute, University of Göttingen, with participating researchers from the University of Hanover. The initiative started October 1st, 2019 with the opening of ten PhD positions and one postdoctoral position.

The RTG Fourier Analysis and Spectral Theory is taking an interdisciplinary and innovative approach to the classical and powerful machinery modern harmonic and Fourier analysis and spectral theory. We focus on its development in the context of mathematical physics, topology and analytic number theory.

A core theme of the RTG is analysis and spectral geometry on Riemannian manifolds, in particular, locally symmetric spaces or more generally spaces acted on by groups. Besides a topological structure, in many interesting cases they also have some arithmetic or combinatorial structure, and one of the key questions involves the fascinating interplay between the spectral properties of certain associated operators on the one hand, and geometric, topological or arithmetic properties on the other. Some prototypical examples of this interaction featured in this RTG are the spectral theory of Cayley graphs of groups; analytic $L^2$-invariants, which link harmonic analysis to topology; and the resolvent and scattering theory of geometric differential operators on singular manifolds. A cornerstone at the interface of modern analytic number theory and harmonic analysis is the theory of automorphic forms, viewed as eigenfunctions of a family of operators on a locally symmetric space. Fourier and harmonic analysis also appear prominently in many applications of classical analytic number theory, in the representation theory of Lie groups and groupoids, and in the construction of quantum field theories with microlocal methods.

On the methodological side we, draw from a variety of analytic techniques, such as microlocal analysis, symbolic calculus, trace formulas and Plancherel theory, Fourier analysis in numerous variations, spectral and scattering theory of operators, but also classical analysis such as a careful analysis of oscillatory integrals.