Econometrics Preparatory Course
Winter Term 2016/2017

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Class time	block course, October 4 – October 10; 14:00–18:00
Location	MZG 8.136 (14:00–16:00); MZG 5.111 (16:00–18:00)
Webpage	see StudIP

Overview and objectives

The econometrics preparatory course aims to refresh the students' knowledge of matrix algebra and basic statistics necessary to succeed in the Econometrics Module. Assuming a diverse student background, the course will guide through several results in linear (matrix) algebra and statistics with a hands-on, data-oriented approach.

Arguably the most important estimator in econometrics is the ordinary least squares (OLS) estimator for the multiple linear regression model:

$$\hat{\beta} = (X'X)^{-1}X'y = (\sum_{i=1}^{n} x_i x_i')^{-1} \sum_{i=1}^{n} x_i y_i. \tag{1}$$

This formula alone contains several concepts from matrix algebra: *transposition*, *inversion*, *scalar* and *vector multiplication*, *matrix summation*. Furthermore, the *matrix* $X'X$ is known to be *symmetric* and *positive semi-definite*. In order to be able to derive (1) from the basic principles of OLS estimation, one needs to be comfortable with *matrix differentiation*.

As it will be shown in Econometrics I, the OLS estimator $\hat{\beta}$ is 1.) linear, 2.) unbiased, and 3.) efficient (in the class of linear and unbiased estimators). More precisely:

1. $\hat{\beta}$ is a *linear function* of y;
2. The *expectation* of the vector $\hat{\beta}$ is the vector β, i.e., $\mathsf{E}(\hat{\beta}) = \beta$.
3. The *variance-covariance matrix* of the vector $\hat{\beta}$ is *smaller* than that of any other linear, unbiased estimator.

A further result in Econometrics I is that, under certain assumptions, the OLS estimator is also the *maximum likelihood* estimator. Under the same assumptions, we can *test* the significance of β_j (an element of the vector β) with the *t-test*, either by calculating the *p-value*, or calculating a *confidence interval* for the desired *significance level*.

The objective of the course is to fill the concepts typeset in *italics* with life and make students comfortable with manipulating potentially high-dimensional matrices and vectors. The course will consist of non-technical lectures featuring small-scale paper-pencil examples, and separate computer sessions where we apply the concepts through manipulating real-world data matrices (X and y) with the R programming language.

Course outline

1. Vectors and n-dimensional spaces: Vectors, manipulations with vectors, orthogonality, linear independence, span, orthogonal complement.

2. Matrix algebra:

    a) Matrices and linear operations with matrices,

    b) Matrix multiplication,

    c) Transpose, trace,

    d) Determinant,

    e) Dimension, rank,

    f) Inverse,

    g) Definiteness, the Löwner ordering,

    h) Differentiation,

    i) Special matrices: square, identity, unit, symmetric, idempotent,

    j) Stochastic matrices: expectation, variance-covariance matrix.

3. Statistics recap:

    a) Maximum likelihood: principle, small illustrative example,

    b) Distributions: Normal, χ^2, F,

    c) Testing: on the example of the t-test, p-value, confidence interval.

Readings

The material of the course is standard and can be found in many standard textbooks. Particularly useful, however, are the following resources:

1. Schmidt, Karsten, and Götz Trenkler: *Moderne Matrix-Algebra*, Springer, 1998.
   Chapters: 1.1–1.5; 2.1–2.6, 2.8; 3.1–3.3; 10.1–10.2.

2. Sydsæter, Knut, Peter Hammond, and Arne Strøm: *Essential Mathematics for Economic Analysis*, Pearson, 2012.
   Chapters: 15.2-15.5, 15.7, 15.8; 16.1–16.2, 16.4, 16.6.

3. Harville, David A.: *Matrix Algebra from a Statistician's Perspective*, Springer, 1997.
   Chapters: 1.1–1.3; 3.1–3.2; 4.4; 5.1–5.3; 8.1–8.2; 13.1–13.2; 15.2, 15.4–15.5.

4. Petersen, Kaare B., and Michael S. Pedersen: *The Matrix Cookbook*, Ver. November 15, 2012.
   Freely available online at: https://www.math.uwaterloo.ca/~hwolkowi/matrixcookbook.pdf

5. Casella, George, and Roger L. Berger: *Statistical Inference*, Cengage Learning, 2001.
   Chapters: 3.3; 7.2.2; 8.1, 8.3.1.